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O.R. Applications

Optimal bidding and contracting strategies for capital-intensive goods

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Abstract

This paper models contracting arrangements between one Seller and one or more Buyers when the "deliverable" or output under the contract is produced in a non-scalable capital-intensive production facility. The basic model proposed allows the Seller and Buyers to negotiate bilateral contracts for the good in advance. On the day, the Seller and Buyers can also sell excess capacity or buy additional non-contract output in an associated backup market, which is being referred to as the spot market for the good. The type of bilateral contract studied has a two-part contract fee structure, and it is at the foundation of practical contracts used by many capital-intensive industries, where capacity can only be expanded well in advance of output requirements. The first part is a reservation cost per unit of capacity, and the second, an execution cost per unit of output when this capacity is actually used. This paper derives the Seller's optimal bidding and Buyers' optimal contracting strategies in a von Stackelberg game with the Seller as the leader. We show that Buyers' optimal reservation level follows an index that combines the Seller's reservation and execution cost. The Seller's optimal strategy is to reveal its variable cost of producing output while extracting its margin from the Buyers using the capacity reservation charge. This structure allows for the Seller to assure in advance its ability to pay the capital costs of capacity while providing Buyers appropriate incentives to take advantage of better terms on the day if alternative, cheaper sources should arise after contracts have been set.

Keywords: Bidding; Contracting; Capital-intensive goods; Investment; Modeling

1. Introduction

This paper considers a market for a capital-intensive and non-storable good or service in which there are a producing Seller and one or multiple Buyer(s). The Seller can either sign long-term

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contracts with Buyers to provide up to a fixed quantity of output ex post, or it can attempt to sell its output in an alternative market, called here the spot market. Similarly, Buyers may sign long-term contracts to cover their anticipated needs, or they may cover these needs through purchases in the spot market. This rather general setting applies to a number of capital-intensive industries, as well as to dated or perishable services, such as electric power, hotels and airline seats. It also applies to supply contracts in which Buyers use only limited amounts of inventory, relying instead on JIT deliveries from their Sellers.

The original motivation for this paper came from the energy sector, which has been going through a period of rapid restructuring from vertically integrated franchise operations to competitive structures in which production, transportation/transmission and distribution are separately priced and typically also provided by different Sellers (see, e.g., [6,10]). The resulting market structure has stimulated an explosive growth in financial and contractual innovations. Foremost among these is the development of bilateral contracts of the sort discussed here. In natural gas, contracts for production and delivery, which used to have a duration of years, are now being bid on a weekly and monthly basis [9]. In electric power, bilateral contracts are at the foundation of many of the evolving models of the restructured market place [3,15,16]. Contracting serves in these markets both the important role of price discovery as well as the obvious direct role of coordinating capacity commitments with anticipated demand. Such bilateral contracts take many forms, but basically they commit a Seller to reserve a certain amount of capacity (e.g., 100 Megawatts of power) during a specific period (e.g., each weekday from 6 a.m. to 10 p.m. during a specific week) for a particular Buyer's use (e.g., a distribution company selling energy to final customers). Additional fees levied if the capacity is actually used (in the example noted, these would be levied per Megawatt-hour delivered). Such contracting takes place against the backstop supply source of the spot markets that provide output "on the day". ("On the day" market can be quite varied. They typically take the form of either a day-ahead options market or a real-time or close to real-time spot market. In the options market, forward options on capacity between Buyers and Sellers are exercised and production is scheduled to run for the next day or the next week. In the real-time market, day-ahead commitments are finetuned through a process of incremental bids and offers. It is clearly important to be clear about which "on the day" market one intends. For concreteness, the reader familiar with electricity markets may think of the spot market studied here as the day-ahead options market, in which generators are informed about a day ahead of which options will be exercised, and generation to meet these is scheduled for the following day.) In natural gas and electric power, such spot markets are well developed, including various futures trading instruments and other derivatives to assist in price discovery. These exist through the New York Mercantile Exchange in the US and through other exchanges throughout Europe, Latin America and Asia.

This paper only deals with a single Seller as would be a reasonable approximation for energy markets in which a single Seller has significant market power by reason of network constraints or regulatory fiat. Even where an industrial customer has strong traditional ties to a local distribution company with local generation assets, any bilateral contracts that might be executed by this customer would clearly be conditioned by the price of alternative sources of energy from existing spot markets or pools from which the customer might be supplied, either by his traditional supplier or by a competitive energy broker/supplier. The model developed here shows how the interplay between the local Seller's cost structure and the distribution of spot prices influences the desirability and level of bilateral contracts likely to be executed. This paper also provides valuable insights on the optimal balance for generator owners (and their traders) between selling capacity in the forward/contract market versus selling to the spot market. (Extensions of this work to a multi-Seller, multi-Buyer setting are developed in [37,41,42] to model more realistically the electric power context, where market power is being undermined by competitive restructuring initiatives. These multi-Seller results rely on the results shown here for the single Seller case.)

Beyond electric power, an important class of applications of the results of this paper arise from capital-intensive production facilities, in which capacity must be planned well in advance of production and in which there are identifiable customers/Buyers for the output of this capacity. Examples include plastics and chemicals where electronic exchanges have been recently established, which serve as spot markets for the products involved. (See the websites for Commerxplasticsnet.com and ChemNet.com, which describe the rapidly evolving spot and contract services being offered electronically.) The contract market in these instances is the contractual procurement market for monthly, quarterly or annual supplies, which provides a forward market for these products. For many specialty chemicals, the forward market frequently involves a single Seller selling under contract to one or more Buyers. While the standard contract today in these markets remains a single-price contract, in contrast to the more general two-part price structure studied here, one can expect further evolution of these markets to see the development of more complex derivative instruments, including those of the form studied here.

The model and framework developed here are also applicable to certain contracting problems in the service sector when the services in question are dated/perishable and produced by capital-intensive service providers. Examples include airline and hotel services. In this case, producing Sellers are airlines or hotels. The Buyers are travel agencies or large customers. Long-term contracts are used by large travel agencies (e.g., for resort hotels) to assure the airlines/hotels of reasonable utilization of their assets, with last-minute demand priced and provided for in the "on the day" market. Note that the capital intensity of the service provider is important since if output is immediately scalable to demand, long-term contracting for capacity would only be of limited interest.

The non-storability aspect of this problem gives rise to the so-called two-goods problem. The first of two goods is the availability of capacity itself, pre-committed to a specific Buyer. The second good is the output actually delivered on the day to the Buyer. When goods are storable, supply chains can be structured with less concern for peak demands and special, unexpected conditions driving demand. These would be met by appropriate inventory and supply chain design strategies (see, e.g., [30]). In the cases considered here, while limited inventories may be physically possible, they are far too expensive to be economically feasible. In the cases considered here, availability of capacity must be pre-arranged (i.e., contracted for) with additional fees paid by the Buyer if the capacity is actually used to produce output on the day. The key issue we study here is how spot pricing and bilateral contracting in these markets are or should be linked. From the Seller's perspective, pre-committing capacity at a fixed price to a particular Buyer may exclude more profitable opportunities through the spot market on the day, if the Seller can guarantee access to this market. The same is true for Buyers signing such precommitment agreements. Thus, as we shall see, the key tradeoff in determining how such contracts should be priced and how much capacity should be committed to them by Sellers and Buyers are the relative costs and risks of sourcing from the contract versus the spot market.

Bidding, investment and contracting strategies are certainly not new with this paper. In the bidding area, an extensive literature exists for both linear and non-linear pricing schedules [33] as well as in the well-developed area of auctions [21–23]. However, as Rothkopf [26] points out, little has been done for repeated auctions/bidding of the type modeled here and characteristic of on-going markets. Investment theory has been ably summarized recently by Dixit and Pindyck in [8]. They emphasize the fundamental importance of the interaction of uncertainty, dynamics and flexibility (or irreversibility) in an options theoretic framework. Similarly, this topic has been addressed in [14,24], but none of this work addresses the two-goods problem nor the managerially important details of optimal contract structure analyzed in this paper. The contracting arrangement developed here assumes that Buyers have access to a competitive spot market, but they do not have (easy) access to a similar competitive market in which they can secure options on this supply. In the standard case [2,20], there is both a competitive spot market as well as a competitive capacity options market. Such an approach is not applicable if the capacity options supply is controlled by a monopolist, the case analyzed here.

In the Operations Management literature, capacity expansion has been analyzed under a number of assumptions, beginning with the seminal work of Alan Manne [19]. Contracting in supply chains has also been the subject of considerable interest in both economic and management science. The economics work has been recently summarized in [4], and the management science work [1,7,17,31] in [30] as well as in [5,13] and [28]. Related management science work [18,25] also considers the issue of dual sourcing (i.e., competitive Sellers), treated in Part II of this research [37,41,42]. This earlier management science work addresses a whole range of contracting issues in supply chain management, typically under conditions of storability. What it does not address is the interaction of capacity and pricing commitments, with an alternative backstop technology (our spot market), which are the essential ingredients of the real problems motivating this research.

The rest of this paper is organized as follows. In Section 2, we consider the case of a single Seller and a single Buyer. We also establish in this section conditions on the cost and access probability of a Seller to participate in the contract market. We note that these conditions (for a given set of Buyers) may be satisfied for only a single Seller even when there are many potential Sellers for the given Buyers. Last, in Section 3, we extend these results to the more general case of a single Seller facing many contracting opportunities with multiple Buyers. We conclude in Section 4 with a number of suggestions for future research.

2. Single Seller, single Buyer

This section models the procurement problem for non-storable goods from two potential sources: long-term contracting and spot market purchases. There are two agents in this model: a Buyer that needs to purchase non-storable goods and a Seller with whom the Buyer can sign long-term contracts. Other Sellers provide additional supply on the day through a spot market for the good. This spot market provides essentially a backstop technology that can be used by Buyers if they do not contract with the Seller. The sole source of uncertainty is the spot market price P_s. This situation is modeled as von Stackelberg game with the Seller as the leader. The Seller selects his profit maximization contract prices (reservation cost s per unit of capacity and execution cost g per unit of output actually used) anticipating how the Buyer will react. Given these costs $[s,g]$, the Buyer determines its optimal reservation or contract level Q. Contracts are signed before the market price P_s is known. After observing the spot price (on the day), the Buyer determines how many units to purchase under the previously executed contract and how many to purchase from the spot market.

We now get into the details of our model. The capacity for the Seller is K. The cost of the Seller is characterized by the parameter pair (b,β), where b is the short-run marginal cost of providing a unit at the Seller's door, and β is unit capacity cost per period. The Seller is assumed to offer long-term contracts to the Buyer in the form of a specified amount of reserved capacity Q and subject to a two-part tariff of the form $[s,g]$.

We denote the Buyer's total demand, on the day, from both contract and spot purchases as $D(P_s(\omega),Q)$. We show below that, as this notation suggests, the Buyer's optimal consumption on the day will depend only on the spot market price $P_s(\omega)$ and the capacity reservation level Q. The spot market price at the state of the world ω is denoted $P_s(\omega)$ and is assumed to be uninfluenced by either the Seller or the Buyer. The distribution of the spot market price is assumed to be common knowledge. (The assumption of a competitive spot market is, of course, only an imperfect approximation for many real problems. For example, there is mounting evidence that many restructured power markets still exhibit considerable market imperfections in the spot market, as discussed by Green and Newbury [11], and others. Interestingly, as pointed out in the recent review by Bower and Bunn [3], there is a growing belief that introducing bilateral trading of the sort envisioned here can partially remedy some of these market power problems. The case of an imperfect spot market, influenced by either the Seller or the Buyers, requires a very detailed assessment of how spot price is affected by various bidding strategies [see, e.g., 11]. We model some imperfections in the spot market in terms of access to it by the Seller, as noted below. But we leave for future research the general case of imperfectly competitive markets. We also do not treat the case here in which Sellers and Buyers face different price distributions in the spot market, a reasonable further generalization.)

Given the Seller's offer $[s, g]$, the Buyer decides how much to contract with the Seller, $Q(s, g)$. On the day, when the state of the world ω is known, the Buyer decides how much to use from this contract, denoted as $q(P_s(\omega))$ and how much to purchase from the spot market, denoted as $x(P_s(\omega))$. The Buyer consumes up to its contracted amount only if the spot market price is higher than its contracted price, i.e., its actual consumption $q(P_s(\omega), g, Q, D)$ satisfies

$$q(P_s(\omega), g, Q, D) = \min[Q, D(P_s(\omega), Q)]\chi(P_s(\omega) - g), \tag{1}$$

where $D = D(P_s(\omega), Q(s, g))$ is the total demand of the Buyer, and $\chi(\cdot)$ is the indicator function (which takes the value of 1 if its argument is positive and 0 else).

The Buyer's residual demand not covered by the long-term contract is covered in an open spot market. This open spot market is supported by the Seller as well as by other Sellers. Given a market price, the Seller will bid its output into the spot market if its short-run marginal cost (b) of supply is less than the price paid him ($P_s(\omega)$). For notational simplicity, we will omit the state variable ω in the following sections.

2.1. The Buyer's problem

In this subsection, we define the Buyer's objective function, provide justifications of underlying assumptions, then derive the Buyer's optimal reservation level Q^*, and finally give some examples.

Using the standard quasi-linear form of utility (e.g., see [32, pp. 97], the Buyer's utility at P_s is given by (Thus, Buyer WTP functions are assumed to depend only on the state of the world through the spot price. This case will allow us to focus on the structure of the optimal solutions without notational complexities.)

$$V(D, Q, P_s) = U(D) - sQ - g\,q(P_s, g, Q, D) - P_s x, \tag{2}$$

where the first term is its willingness-to-pay (WTP) at P_s, evaluated at the realized demand $D(P_s, Q)$, the second and the third term together are the payment for the goods delivered under the long-term contract, and the fourth term is the payment for goods x purchased in the spot market.

Concerning the form of the Buyer's "utility", this is standard in both economics and marketing science [12,29,32]. Note, in particular, that the WTP function U is the basis for normal demand theory since the solution, call it $D_s(P_s)$, to maximizing $U(D) - P_s D$ is simply $U'(D) = P_s$ or $D_s(P_s) = U'^{-1}(P_s)$. Thus, the standard demand curve for the Buyer is just $D_s = U'^{-1}$ where $U'^{-1}(P_s)$ is the inverse of U'. The standard tools of Marketing Science would be used to specify and estimate the WTP function U (typically a Buyer is a demand aggregator, and so the demand curve for the Buyer is itself a composite of underlying final demand across the customer segments it serves. Thus, mark-up strategies and service differentiators at the retail level will affect the Buyer's WTP/demand functions U and D. These service and "brand" characteristics of the Buyer would be included in the assessment of the demand curve as seen by the upstream Seller, as per established marketing research approaches to demand analysis. See, e.g., [12] and, for vertical price management between Sellers and Buyers, Chapter 9 in [29]). Since we are considering here dual sources of fulfillment (namely, spot or contract purchases), a more complicated solution for the demand equation results than the traditional single-source demand function $D_s(P_s)$. Nonetheless, as we now show, the dual-source demand function of interest here is a transformation of $D_s(P_s)$, so that the standard demand curve $D_s(P_s)$ is the basic building block of the results presented here.

Using (1), (2) can be rewritten as

$$V(D,Q,P_s) = U(D) - P_s D + (P_s - g)^+ \times \min[D,Q] - sQ. \quad (3)$$

The Buyer's problem can be solved in two stages. Starting at the end (the first-stage), once P_s is known and Q is set, the Buyer solves for q and x. Knowing these solutions, the Buyer can solve for Q (the second-stage) by maximizing the expected value of $V(D,Q,P_s)$ in (3). In keeping with decreasing marginal utility of consumption (note that this does not imply anything about risk preferences. The Buyer is in fact risk neutral in our model since $V(\cdot)$ is linear in money. The consequences of risk aversion by the Buyer are briefly indicated below. Decreasing marginal utility consumption simply means the Buyer's traditional demand curve D_s is downward sloping, since as noted earlier $D_s = U'^{-1}$), we will assume that the Buyer's WTP $U(z)$ is strictly concave and increasing so that

$$U'(z) > 0, \quad U''(z) < 0, \text{ for } z \geqslant 0. \quad (4)$$

In the first-stage, $D(P_s,Q)$ is given as the solution to:

$$\text{Maximize}_{D \geqslant 0}\, V(D,Q,P_s) = \text{Maximize}_{D \geqslant 0}\, [U(D) - P_s D + (P_s - g)^+ \min[D,Q] - sQ]. \quad (5)$$

Lemma 1 characterizes the Buyer's demand curve when facing both a spot market as well as a contract market.

Lemma 1.

$$D(P_s,Q) = \max[D_s(P_s),Q] = \max[U'^{-1}(P_s),Q]. \quad (6)$$

Proof. See Appendix A. □

Lemma 1 indicates that when the Buyer has dual sources for its procurement, its demand curve would change from downward-sloping (single source) to be kinked (dual source). Whether to use contract purchases or to use spot market purchases, the Buyer's first-stage decision, depends entirely on the comparison of spot market price P_s on the day with the Buyer's marginal WTP $U'(Q)$: if $P_s < U'(Q)$, then $q = 0$, i.e., contract purchases dominated by spot purchases, and this is true even though the Buyer may have reserved capacity $Q(s,g)$ with the Seller. Otherwise if $P_s > U'(Q)$, then $x = 0$, i.e., contract purchases dominate spot purchases. As noted in the proof of Lemma 1, the Buyer will never contract for any greater amount than it would find reasonable to use at the contract execution price g, so that, in fact, $g \leqslant U'(Q)$ (or equivalently $Q \leqslant D_s(g)$) must hold for any reasonable Q. Lemma 1 is only defined for such "reasonable" values of Q. We discuss further below the implications of Lemma 1 for optimal q and x. In any case, total demand is derived from the basic building block of the Buyer's standard demand curve $D_s(P_s)$.

Lemma 1 gives the optimal consumption portfolio of q and x. Given Lemma 1, (1) can be simplified as

$$q = \begin{cases} Q & P_s > g, \\ 0 & \text{otherwise,} \end{cases}$$

which indicates that the optimal decision on contract use is a rather simple bang-bang solution: If $P_s > g$, use all reserved capacity Q, otherwise do not use any reserved capacity.

For spot market purchase, x can be simplified as

$$x = \begin{cases} D_s(P_s) & P_s < g, \\ D_s(P_s) - Q & g < P_s < U'(Q), \\ 0 & P_s \geqslant U'(Q), \end{cases}$$

which indicates that the optimal decision on purchase is to check whether P_s falls in or out of the range of $[g, U'(Q)]$. If it is out of the range but $P_s \geqslant U'(Q)$, purchase nothing from the spot market; else if $P_s < g$, purchase only from the spot market. If P_s falls in the range, then fulfill only the residual demand from the spot market by purchasing $D_s(P_s) - Q$.

The second-stage of the Buyer's decision is to find a reservation level Q in order to maximize its expected utility $EV(Q)$, i.e.,

$$\text{Maximize}_{Q\geqslant 0} EV(Q) = \int V(D,Q,P_s) f(P_s)\,\mathrm{d}P_s, \tag{7}$$

where $D = D(P_s, Q)$ is given by (6), and where $f(P_s)$ is the probability density function of spot market price P_s, assumed to be common knowledge among all market participants.

We now define the "effective price function" ("G" function for short; we will explain its meaning below) $G(p)$ as

$$G(p) = \int_0^p (1-F(y))\,\mathrm{d}y = \int_0^p P_s f(P_s)\,\mathrm{d}P_s + p(1-F(p)) = E[\min(P_s,p)], \tag{8}$$

where $F(y)$ is the cumulative distribution function of the spot market price P_s, $\mu = E\{P_s\}$ is the mean of the spot market price. The second equality is derived using integration by parts. Notice that,

$$\lim_{p\to\infty} G(p) = \lim_{p\to\infty} \int_0^p (1-F(y))\,\mathrm{d}y = \mu, \tag{9}$$

and for any real number z,

$$E(P_s - z)^+ + E[\min(P_s, z)] = \mu. \tag{10}$$

The meaning of the "G" function is the following. By (9), $G(p)$ equals the expected spot price as p approaches infinity. For finite p, by (8), $G(p)$ is just the expected value of $\min(P_s,p)$. If a contract can be executed at price p, then this represents the expected price per unit paid by a Buyer, since the Buyer will elect to use the spot price P_s when this is less than p and will use the contract when p is less than the spot price. Thus, the terminology for the G function as "effective price function".

Lemma 2. *$EV(Q)$ is strictly concave in Q, $\forall(s,g)$.*

Proof. This follows from

$$\frac{\partial^2 EV}{\partial Q^2} = G'(U'(Q))U''(Q) = (1-F(U'(Q)))U''(Q) < 0. \qquad \square$$

Denote $G^{-1}(\cdot)$ as the inverse function of $G(\cdot)$ and define $z^+ = \max(z,0)$. Denote $Q^*(s,g)$ as the optimal reservation level for the Buyer.

Theorem 1 (Buyer's optimal contracting policy). *Assume* $\lim_{x\to 0^+} U'(x) < \infty$. *When*

$s + G(g) > G(U'(0)),\ Q^*(s,g) = 0.$

Otherwise, $Q^*(s,g)$ *is determined by any of the following equivalent identities*:

(i) $Q^*(s,g) = D_s(G^{-1}(s+G(g)))$,

(ii) $E[\min(P_s, U'(Q^*(s,g)))] = s + E[\min(P_s, g)]$,

(iii) $E(P_s - U'(Q^*(s,g)))^+ = s + E(P_s - g)^+$,

(iv) $U'(Q^*) = s + g + \int_g^{U'(Q^*)} F(P_s)\,\mathrm{d}P_s$,

(v) $G(U'(Q^*)) - G(g) = s$.

Proof. When $s + G(g) > G(U'(0))$, the optimal contracting strategy for the Buyer solving (7) is $Q^*(s,g) = 0$. Now we consider the case of $s + G(g) \leqslant G(U'(0))$. Using (3) we write the expected utility $EV(Q)$ as

$$EV(Q) = -sQ + \int_g^\infty (P_s - g)\min[Q,D] f(P_s)\,\mathrm{d}P_s + \int_0^\infty [U(D) - P_s D] f(P_s)\,\mathrm{d}P_s. \tag{11}$$

From Lemma 1, (11) can be written as

$$EV(D(Q),Q) = -sQ + \int_g^\infty (P_s - g)Q\,\mathrm{d}F(P_s) + \int_0^{U'(Q)} [U(D_s(P_s)) - P_s D_s(P_s)]\,\mathrm{d}F(P_s) + \int_{U'(Q)}^\infty [U(Q) - P_s Q]\,\mathrm{d}F(P_s).$$

Then the first-order condition gives

$$\begin{aligned}\frac{\partial EV}{\partial Q} &= -s + \int_g^{\infty} (P_s - g)\,\mathrm{d}F(P_s) \\ &\quad + \int_{U'(Q)}^{\infty} (U'(Q) - P_s)\,\mathrm{d}F(P_s) \\ &= -s - \int_0^g P_s\,\mathrm{d}F(P_s) - g[1 - F(g)] \\ &\quad + \int_0^{U'(Q)} P_s\,\mathrm{d}F(P_s) + U'(Q) \\ &\quad \times [1 - F(U'(Q))] \\ &= -s - G(g) + G(U'(Q)) = 0. \end{aligned} \tag{12}$$

The first equality of (12) is due to (8), the second equality (FOC) gives identity (v) in Theorem 1. Identities (i), (ii) and (iv) are direct consequences of identity (v) by using the definition of G in (8). Using (10), we obtain identity (iii). Hence the proof. □

The intuition underlying Theorem 1 is as follows. If the combined effective price per unit of capacity $(s + G(g))$ exceeds the threshold level of $G(U'(0))$, the Buyer will not contract for any capacity at all, but will rely entirely on the spot market for supply. The upper bound $s + G(g)$ represents the sum of the cost per unit to reserve capacity s plus the expected cost of using this capacity on the day $G(g)$. When this combined effective price exceeds the maximum effective price the Buyer is prepared to pay for any unit of capacity (recall that $U'(Q)$ is decreasing in Q), then it is better to rely on the spot market for all purchases.

Concerning the case of positive contracting, consider for example identity (iv). The left-hand side is the Buyer's marginal WTP at the contracted capacity Q^*. The right-hand side is the sum of the marginal cost of the final unit reserved and consumed plus the marginal insurance benefits from the contract relative to the realized spot market price.

This tradeoff between the marginal cost of additional reserved capacity and hedging benefits from contracting can be seen alternatively in identity (v). We see from this that the cost of an additional unit of reserved capacity is equated to the total benefits of the additional unit in those states of the world in which the contract will be executed, namely, for $g \leqslant P_s(\omega) \leqslant U'(Q^*)$.

Corollary 1. *Q^* is monotonically decreasing in s and g, $\forall Q^* > 0$. Moreover, when $Q^* > 0$,*

$$\frac{\partial Q^*}{\partial g} = (1 - F(g))\frac{\partial Q^*}{\partial s}. \tag{13}$$

Proof. Taking the derivatives w.r.t. s and g in identity (4) in Theorem 1,

$$\frac{\partial Q^*}{\partial s} = \frac{1}{U''(Q^*)[1 - F(U'(Q^*))]} < 0,$$
$$\frac{\partial Q^*}{\partial g} = \frac{1 - F(g)}{U''(Q^*)[1 - F(U'(Q^*))]} < 0.$$

The above two equalities imply (13). □

Corollary 1 embodies the expected results that the Buyer contracts less if the Seller charges a higher subscription fee (s increases) or a higher contracting price (g increases). Notice that (13) characterizes the fundamental relationship between the Seller's optimal reservation charge s and its optimal execution charge g. The left-hand side of this expression is the change in the optimal contract amount $Q(s,g)$ as g changes, while the right-hand side is change in $Q(s,g)$ as s changes, weighted by the probability that the spot price will exceed g, which is precisely when the contract will be exercised by the Buyer.

Practically, the results in Theorem 1 can be used to support the decision-making of Buyer managers both qualitatively and quantitatively. In deciding whether to sign a long-term contract with the Seller, the Buyer first determines the function $s + G(g) = G(U'(0))$ which indicates the region in which contracting will and will not be profitable (see Fig. 1). If the Seller's bid price pair (s,g) lies in the region above this curve, then no contract is optimal; otherwise, $Q^*(s,g)$ as determined by Theorem 1 specifies the optimal quantity to sign with the Seller. We now give two examples, one numerical and the other analytical, to illustrate Theorem 1.

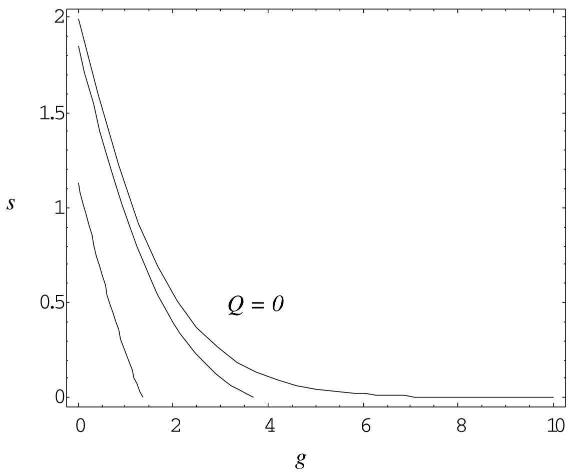

Fig. 1. Iso-quants of the Buyer's optimal contracting strategy, solving $s = G(U'(Q)) - G(g)$. (From top to bottom) $Q = 0, 1, 2$.

Example 1 (*Numerical*). Suppose P_s follows the Erlang (Γ) distribution $f(x) = 1/(\Gamma(2))x\mathrm{e}^{-x}$ and $U(x) = 10(1 - \mathrm{e}^{-10x})$. Fig. 1 shows two optimal contracting strategies (for iso-quants $Q = 0, 1, 2$) for various offers from the Seller. Fig. 2 (3) shows the sensitivity of $Q^*(s, g)$ to s (g) for various levels of g (s).

Example 2 (*Analytical*). Assume the spot market price is uniformly distributed between P_d and P_u, and the Buyer's WTP has the form $U(x) = \alpha/\gamma(1 - \mathrm{e}^{-\gamma x})$. Assume

$$g < \min\left[\alpha, P_u - \sqrt{2s(P_u - P_d)}\right] \quad \text{and} \quad s < \alpha, \tag{14}$$

then the Buyer's optimal contracting strategy is

$$Q^*(s,g) = \frac{1}{\gamma}\ln\frac{\alpha}{P_u - \sqrt{(P_u - g)^2 - 4\sqrt{3}s\sigma}}, \tag{15}$$

where σ, the standard deviation of the spot market price, is $\sigma = (P_u - P_d)/(2\sqrt{3})$. If either of the inequalities is violated in (14), then $Q^*(s, g) = 0$.

2.2. The Seller's problem

Henceforth, we will take the Buyer's contracting strategy $Q(s, g)$ to be the optimal strategy $Q^*(s, g)$ given in Theorem 1. When no confusion is likely, we will omit the superscript "*". Given this contracting strategy, the Seller is assumed to determine its optimal bidding strategy $[s^*, g^*]$ that maximizes its expected profit. In this subsection, we first derive the Seller's profit function, then we derive Seller's profit maximization strategies anticipating the Buyer's reaction, last, we derive the condition for the existence of such an equilibrium for which $Q > 0$ and give one example. The profit function $\prod(s, g; P_s, Q(s, g))$ for the Seller at P_s from

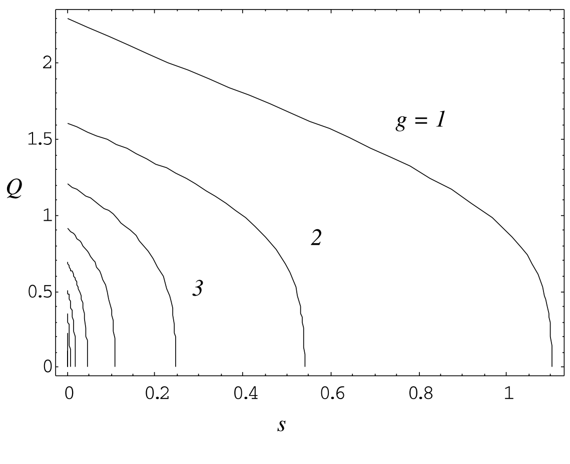

Fig. 2. The sensitivity of $Q(s, g)$ to s for $g = 1, 2, 3, 4, 5, 6, 7, 8, 9, 9.9$, from top to bottom.

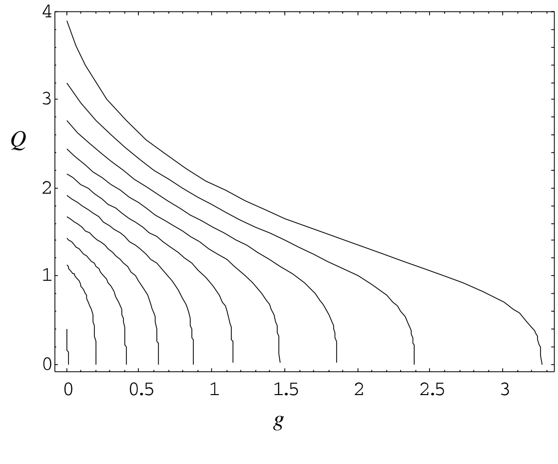

Fig. 3. The sensitivity of $Q(s, g)$ to g for $s = 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 1.99$, from top to bottom.

the long-term contract and from the spot market is given as

$$\prod(s,g;P_s,Q(s,g)) = sQ(s,g) + gq - (\beta K + bq) + m(P_s - b)^+(K - q). \tag{16}$$

The first two terms on the right-hand side of Eq. (16) represent the Seller's revenue from the contract, the third term is the Seller's cost of supplying q units to the Buyer and the fourth term is the Seller's profit from the spot market (where the Seller will only sell if $P_s \geqslant b$). We assume that on the day the Seller can only sell a percentage $(0 \leqslant m \leqslant 1)$ of its residual output in the spot market. Alternatively, m may be thought of as the probability that the Seller will find an appropriate customer for selling residual output on the day. The risk factor m, which is assumed to be fixed here, provides the incentive for the Seller to sign a contract with the Buyer.

Concerning the risk factor m, this would in practice be estimated from historical data, leading in stable markets to rational expectations about its value, which typically will vary from one Seller to another. There are several reasons why this factor may be less than unity. The spot market may be "thin" or not well organized, making it difficult to identify Buyers on the day. There may be difficulties in arranging transportation, because of transportation capacity constraints or thinness in the transportation market. This latter case is the situation, for example, in many energy markets in which the transportation (or transmission) function is fraught with congestion and uncertainty. We will see below (Corollary 4) that when $m = 1$, there will be no long-term contracting in equilibrium; all transactions will occur through the spot market. Thus, as m goes to unity, long-term contracts disappear. While we do not model the dynamics of the contracting process, it is natural to think of m as decreasing as the spot market becomes more liquid and contracting becomes more short term. These are exactly the changes noted in [9] in the natural gas market. As the short-term contracting market picked up steam in the late 1980s, more and more of producers' capacity was contracted for in the short-term market, eventually driving the long-term (multi-year) contract market transactions to near zero. Quarterly and monthly contracting is now the rule of the day, coupled with various short-term risk management instruments. Presumably the reason that such contracting is not further displaced by the (very liquid) spot market in natural gas is that producers are not certain that they can find Buyers on the day for all their output (i.e., $m < 1$), just as modeled here. Of course, the forward market for natural gas is not just a substitute for the spot market; it also plays a valuable role in promoting price discovery and in allowing gas distributors to be able to provide appropriate risk buffering for their customers. Similar trends are visible in electricity markets. In those power markets where physical bilateral contracting is allowed, it is quite typical for a significant fraction of wholesale energy traded to occur in the bilateral market, with the short-term options markets and spot market acting as fine-tuning mechanism for the contract market a day ahead or on the day. The existence of such active contracting markets bespeaks significant benefits from advanced planning and scheduling of generation-transmission-distribution assets to meet projected demands. (We note in passing that it is straightforward to generalize the results here to the case in which m depends on the spot market price, i.e., $m = m(P_s)$, see [41,42]. Such a dependency is likely to be present, e.g., in electric power markets, since high demand intensity, and associated high spot prices, typically leads to high congestion which leads to access problems.)

Alternative explanations for contracting, besides the risk factor m, could be that the Seller is risk-averse and hedges by selling some at the current prices and the Buyer agrees to reserve this capacity by the same token. Sellers attempt to assure reasonable utilization of their capacity through bilateral contracts and Buyers, who face uncertainty in the spot market, attempt to hedge against price volatility through the same bilateral contracts. Thus, risk-averse behavior of either party would further motivate them to use contracting rather than rely solely on the spot market.

The Seller's decision problem is the following:

$$
\begin{aligned}
\text{Maximize}_{(s,g)} E \prod (s,g) &= \int \prod (s,g;P_s,Q(s,g)) f(P_s)\,\mathrm{d}P_s \\
&= [s + (1 - m\chi(g-b))(g-b)(1-F(g)) \\
&\quad - m(\mu - G(\max(g,b)))]\,Q \\
&\quad + mK(\mu - G(b)) - \beta K, \qquad (17)
\end{aligned}
$$

subject to the constraint that $Q(s,g) \leqslant K$.

Lemma 3 (Optimal execution fee bid by the seller). $g^* = b$.

Proof. From Theorem 1, we know that the Buyer's optimal contract $Q(s,g)$ will remain unchanged as long as $s + G(g)$ remains constant. To prove Lemma 3, we therefore show that for any fixed value of $s + G(g)$, the optimal solution for g is b. To see this, note that along any iso-quant of $s + G(g) = c$, we must have:

$$\frac{\mathrm{d}s}{\mathrm{d}g} = -\frac{\mathrm{d}G(g)}{\mathrm{d}g} = -(1 - F(g)). \qquad (18)$$

Thus, assuming a constant level of contract capacity $Q(s,g)$ along the iso-quant $s + G(g) = c$, we compute from (18) and (17) that

$$
\begin{aligned}
\frac{\mathrm{d}E\prod}{\mathrm{d}g} &= \frac{\partial E\prod}{\partial g} - (1 - F(g))\frac{\partial E\prod}{\partial s} \\
&= \begin{cases} -(g-b)f(g)Q, & g \leqslant b, \\ -(1-m)(g-b)f(g)Q, & g \geqslant b. \end{cases}
\end{aligned}
$$

Therefore, the FOC implies $g^* = b$. □.

We are now in a position to derive market equilibrium prices in the long-term contract market. Define $\epsilon_s(Q)$ as the demand elasticity w.r.t. to the subscription charge s,

$$\epsilon_s(Q) = \frac{s}{Q}\left|\frac{\partial Q}{\partial s}\right| = -\frac{s}{Q}\frac{\partial Q}{\partial s}.$$

Theorem 2 (Seller's optimal bidding policy). *Assume* $QD''_{\mathrm{s}}(Q) + 2D'_{\mathrm{s}}(Q) \leqslant 0$, *then the optimal bidding policy for the Seller is*:

$$
s^* = \begin{cases} \frac{m(\mu - G(b))}{1 - 1/(\epsilon_s(Q))} = \frac{mE(P_{\mathrm{s}} - b)^+}{1 - 1/(\epsilon_s(Q))} & Q < K, \\ G(U'(K)) - G(b) & Q \geqslant K, \end{cases}
$$

$$g^* = b.$$

Proof. See Appendix B. □

The conclusions of Theorem 2 are derived in standard fashion from first-order conditions. It should be noted that the rather technical condition assumed in Theorem 2 (viz. $QD''_{\mathrm{s}}(Q) + 2D'_{\mathrm{s}}(Q) \leqslant 0$) is shown in Appendix B to be a sufficient condition for the global optimality of the expected profit function reached at (s^*, g^*). This condition is satisfied, for example, if the Buyer's normal demand curve $D_{\mathrm{s}}(p) = U'^{-1}(p)$ is concave, including linear. (These are also rather standard assumptions in the finance literature on demand, e.g., [27]; for further discussion and interpretation of these conditions, see [37,38].) This type of condition on the demand function is typical for assuring concavity of expected profits, but weaker sufficient conditions for optimality of the first-order conditions likely exist (as they do for the traditional one-good case). Naturally, as with the usual FOC approach, in cases where this sufficient condition is not satisfied, other global optimization approaches can still be applied to the well-structured problem of maximizing (17). Since (17) is continuous in s, g, and since the optimal s is both non-negative and can be bounded above by Theorem 1 (which shows that Buyer demand is 0 unless $s + G(g) \leqslant G(U'(0))$), we see that this problem (17) has a solution in any case.

Theorem 2 indicates that the optimal bidding strategy for the Seller is to use the reservation charge s to extract margin from the Buyer, setting the execution cost g as low as possible consistent with recovering unit variable operating costs b. The inverse elasticity term in the expression for s^* reflects the usual tradeoff between price (positive with increasing price) and quantity (negative with increasing price) effects in pricing any good (see, e.g., [32]). The optimal reservation charge depends on the unit opportunity cost $(m\mathrm{E}(P_{\mathrm{s}} - b)^+)$ the Seller loses from the spot market due to capacity commitment to the Buyer as well as the inverse elasticity of the Buyers' contract demand (reservation level). The key here is that margin is collected through the reservation charge, essentially because increasing g above its minimum level b decreases the anticipated value of the contract to the Buyer at a faster rate than increases in s. This is so because the Buyer knows that it will only execute the contract on the day in those states of the world where P_s exceeds g so increases in g increase both the Buyer's cost of execution as well as the utility of the contract, as a consequence of the Buyer's behavior, and because of (13), any execution charge g deviating from the unit variable operating costs b decreases the Seller's profit, as rigorously analyzed in the proof of Lemma 3.

Denote Q as the Seller–Buyer joint equilibrium.

Corollary 2 (Seller–Buyer joint equilibrium). *Q is implicitly determined by*:

$$\begin{aligned} &QU''(Q)[1-F(U'(Q))]+G(U'(Q)) \\ &\quad -m\mu-(1-m)G(b)=0. \end{aligned} \tag{19}$$

Proof. This is a direct consequence of Theorems 1 and 2. □

Denote K^* as the optimal Seller–Buyer joint investment policy.

Corollary 3 (Seller–Buyer joint investment policy). *Assume $\beta > m(\mu - G(b))$, then*

$$\begin{aligned} K^* &= D_s\left(G^{-1}\left(\frac{\beta}{1-1/(\epsilon_s(K^*))}+G(b)\right)\right) \\ &= Q(s^*, g^*), \end{aligned}$$

where the optimal bidding policy for the Seller is,

$$s^* = \frac{\beta}{1-1/(\epsilon_s(K^*))}, \qquad g^* = b.$$

Proof. This is a direct consequence of Theorem 2. □

We see from Corollary 3 that when capacity is variable, contracting strategies will determine the optimal capacity when the unit capacity cost is not so small as to make output easily scalable to demand. The key condition on capacity cost is related to the expected margin obtainable per unit of capacity as well as to the probability m of being able to collect this margin on the day. The role of the spot market access probability m is further clarified in the following Theorem 3, where we show that it is fundamental in determining when Seller–Buyer contracting can exist in equilibrium, i.e., when at the Seller–Buyer contract amount as determined by Corollary 2 is positive ($Q > 0$).

Theorem 3 (Condition for the existence of positive Seller–Buyer contract). *Given Q as determined by* (19). *Then $Q > 0$ implies*

$$0 \leqslant m < \frac{G(U'(0)) - G(b)}{\mu - G(b)} < 1.$$

Proof. From Theorem 2, we know that in order for the Seller to sign any long-term contract with the Buyer, its revenue from the contract market s^*Q must offset its opportunity cost at the spot market $mQ(\mu - G(b))$, i.e., $s^*Q \geqslant mQ(\mu - G(b))$, which gives us

$$\begin{aligned} m &\leqslant \frac{s^*}{\mu - G(b)} = \frac{G(U'(Q)) - G(b)}{\mu - G(b)} \\ &\leqslant \frac{G(U'(0)) - G(b)}{\mu - G(b)} < 1. \end{aligned}$$

The above equality holds due to Theorem 1. The second inequality holds due to the decreasing of U' function ($U'' < 0$) and the increasing of G function ($G'(x) = 1 - F(x) > 0$). The last inequality holds due to (9). Hence the proof. □.

Corollary 4 (Riskless spot market for the Seller). *If $m = 1$ then $Q = 0$.*

Proof. This is a direct consequence of Theorem 3 since $m = 1$ violates the necessary condition given in Theorem 3. □

Corollary 4 says that when the Seller incurs no risk on the spot market ($m = 1$), i.e., it can sell all its residual output in the spot market, then the

Seller will not sign any contract with the Buyer ($Q = 0$). The impact of the risk factor m is further elucidated in the following example.

Example 3 (*Risky spot market for the Seller*). Suppose the spot market price distribution and the Buyer's WTP function are the same as in Example 1. Assume $b = 5,\ \beta = 0,\ K = 1$.

1. $m = 0$, the Seller faces an extremely risky spot market. Its profit relies entirely on the contract market, with the optimal contract amount $Q^* = 0.440$. The Seller charges $(s^*, g^*) = (0.0336, 5)$, and obtains an optimal expected total profit of $\prod_{\max} = 0.0148$.
2. $m = 0.1$, the Seller faces a less risky spot market; the slight increase in the optimal reservation fee reflects this advantage, $(s^*, g^*) = (0.0349, 5)$, resulting in less contract demand from the Buyer, $Q^* = 0.424$. However, the Seller's overall profit increases slightly to $\prod_{\max} = 0.0175$. This is so because although his contract profit decreases a bit (to 0.0128), he makes additional profit from the spot market (0.0047).

Example 3 shows how the changing of m will change the behavior of the Buyer as well as the Seller. We see that as m increases the Seller faces less risk in being able to gain access to the spot market. Hence, he would demand more by charging a higher reservation fee to offset the increasing opportunity loss from not waiting to trade in the spot market. As a consequence, the Buyer would reserve/contract less from the Seller as m increases. The Seller's profit in the contract market decreases slightly as a result, but it more than makes up for this from increased earnings in the spot market; hence overall profit increases. Fig. 4 shows the sensitivity of the von Stackelberg leader–follower equilibrium Q to the risk factor m. The contract size has its maximum when the Seller faces an extremely risky market ($m = 0$). It becomes smaller when the market risk decreases. When the risk factor reaches its upper bound, as specified in Theorem 3,

$$\frac{G(U'(0)) - G(b)}{\mu - G(b)} = 0.988,$$

the contract market vanishes and the Seller will not sign any contract with the Buyer. Fig. 5 (6) shows the variation of the Seller's profit $\prod(s, g)$ from both the contract and spot market as s (respectively, g) varies, keeping $g = b$ (respectively, $s = s^*$) fixed.

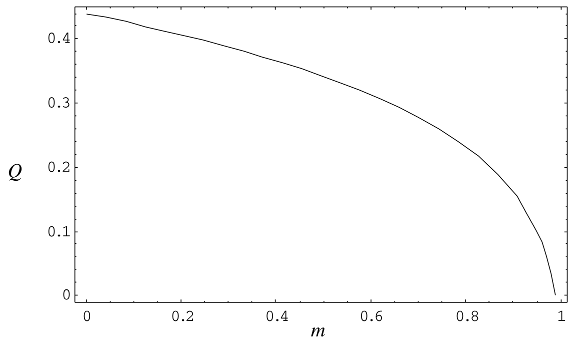

Fig. 4. The sensitivity of Q to m.

Lemma 4 (Conditions for both parties to trade). *Let $g = b$. Then the conditions for both the Seller and the Buyer to profit from trade are the following*:

$$\begin{aligned} \mu - G(b) &\geqslant G(U'(0)) - G(b) \\ &\stackrel{\text{def}}{=} \bar{s} \geqslant s^* \geqslant \underline{s} \\ &\stackrel{\text{def}}{=} \mathrm{E}\{\mathrm{m}(\mathrm{P_s} - \mathrm{b})^+\} \\ &= \mathrm{m}(\mu - \mathrm{G(b)}) \geqslant \mathrm{m}\mu - \mathrm{G(b)}. \end{aligned}$$

Moreover (when $g = b$), the Seller will find it in his interest to participate in the contract market at any reservation fee $s \geqslant \underline{s}$.

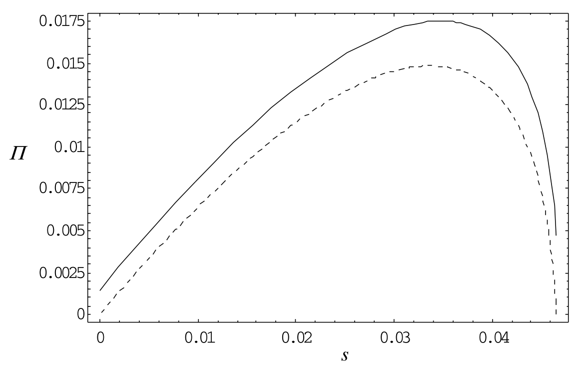

Fig. 5. The variation of the Seller's overall profit with s for fixed $g = b$: (dashed line) $m = 0$; (solid line) $m = 0.1$.

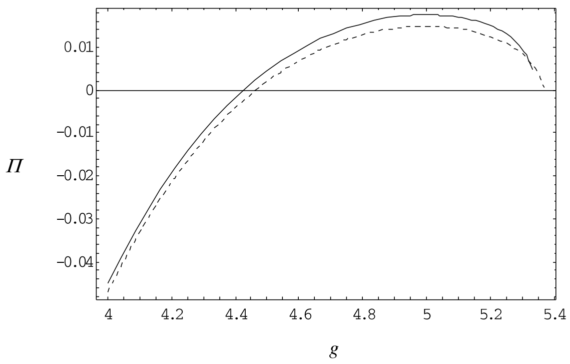

Fig. 6. The variation of the Seller's overall profit with g for fixed $s = s^*$: (dashed line) $m = 0$; (solid line) $m = 0.1$.

Proof. First, since from (8), $\mu \geqslant G(p),\ \forall p \geqslant 0$, we note that the maximum value of s, is a direct consequence of identity (v) of Theorem 1 and i.e., $s + G(b) \leqslant G(U'(0)) \leqslant \mu$. We now derive the minimum value of s. Since from Lemma 3, $g = b$, the expected profit of Seller is given as

$$E\prod(s,b,K) = (s - E\{m(P_s - b)^+\})Q + (E\{m(P_s - b)^+\} - \beta)K. \qquad (20)$$

We see from (20) that no contracting ($Q = 0$) would be preferable to any positive contract unless expected contract revenues are superior (per unit of capacity committed) to the expected revenues available on the spot market, i.e., unless $s \geqslant E\{m(P_s - b)^+\} = m(\mu - G(b))$. The same logic establishes (assuming $g = b$) that the Seller will earn at least as much from each unit of capacity subscribed by a Buyer in the contract market as he would earn by offering the same unit in the spot market, as long as $s \geqslant \underline{s}$. □

It is interesting to notice that the conventional wisdom of $s + G(g) = E\{P_s\} = \mu$ or $s + g = \mu$ or "option fee + execution fee = expected value of the spot price" does not hold as shown in Lemma 4. (We thank the referees for directing our attention to this, leading to the results in Lemma 4 and Corollaries 5–7.) The rationale for Lemma 4 is that the Seller does not want to make any less per unit of capacity than it could by committing to sell its capacity into the spot market (although access to the spot market is imperfect and occurs on the day only with probability m). Equating the two possibilities leads to the lower bound of s in Lemma 4. From Lemma 4, we know that when there is a long-term contract market, the Seller could still be used even if $\beta > E\{m(P_s - b)^+\}$, but the Seller may or may not break even if β is sufficiently large, though participation in the contract market, when it is appropriate, will lead in any case to increased profitability. Recall that we are dealing with the short-run problem here in which capacities are not variable, so that negative profits are indeed possible if investments are sunk and capital costs sufficiently high relative to what the contract and spot market will bear. Note also from Lemma 4 that if m is with very high probability near unity, then the Seller will also face diminished incentives to contract, since the spot market then provides a viable alternative to contracting. On the other hand, index $\underline{s} + G(b)$ explains why other Sellers may not want to enter into the forward market, selling instead their entire output on the spot market. This could very well happen if these Sellers had inferior technologies or better spot market access (e.g., larger b and larger m) than the Seller we model here. Our Seller is willing to trade in the contract market if he has a sufficiently small b and/or low m, resulting in a sufficiently small minimum $\underline{s} + G(b)$. For other Sellers who have higher $\underline{s} + G(b)$, it is in their own interests to stay in the spot market rather than competing in the forward market. The implications of Lemma 4 coupled with the following Corollaries 5, 6 are that the conditions for both Sellers and Buyers who want to trade are quite limited if the Sellers have a good access to the spot market (i.e., m is big enough). This may explain why it seems so difficult to establish effective forward markets for electricity (as pointed out by the referee, for which we are thankful). On the other hand, when the Sellers have significant market access risks, they may well face significant incentives to be active in the forward contract market. (In [37], Wu, Kleindorfer and Sun completely characterize for the multi-Seller and multi-Buyer case the cost, demand and access conditions that lead various types of Sellers to be active in the contract and spot markets, both in the short run with capacity fixed as well as in the long run where capacity is variable.)

The reason the Seller finds participating in the contract market attractive at any $s \geqslant \underline{s}$ is that the contractor has nothing to lose at such a reservation fee. If any units of the contract are purchased at $(s, g) \geqslant (\underline{s}, b)$, then the Seller earns at least as much from such units as in the spot market. Whatever portion of the bid capacity is not purchased in the contract market can always be sold on the spot market, incurring precisely the same risk as if the capacity had not been offered in the contract market. The reason is that we assume the Seller can always sell unused capacity in the spot market (subject to market access risk, which is not affected by the amount bid or sold in the contract market).

Corollary 5 (Impact of mean-preserving changes in the spot market distribution on feasible trade conditions). *Assume fixed* $E\{P_s\} = \mu$ *and let* σ *be some other parameter, e.g., the standard deviation, of the spot price distribution, so that we can write the c.d.f. of* P_s *as* $F(y; \mu, \sigma)$ *and the* G *function in* (8) *as* $G(p; \mu, \sigma)$.

If

$$m < 1 - \frac{\partial G(U'(0))/\partial\sigma}{\partial G(b)/\partial\sigma}$$

then

$$\frac{\partial(\bar{s} - \underline{s})}{\partial\sigma} > 0,$$

else if

$$m \geqslant 1 - \frac{\partial G(U'(0))/\partial\sigma}{\partial G(b)/\partial\sigma}$$

then

$$\frac{\partial(\bar{s} - \underline{s})}{\partial\sigma} \leqslant 0.$$

Proof. This is a direct consequence of Lemma 4. □

Corollary 5 implies that, holding the mean of the spot price distribution fixed, the spread of feasible trade conditions $(\bar{s} - \underline{s})$ could shrink or move further apart as other distribution parameters change, depending on the Seller's market access m. For example, higher volatility alone may not necessarily increase the incentive for both parties to contract in the forward market; the outcome depends on m. We now further examine the impact of market access risk m on trade conditions.

Corollary 6 (Impact of market access risk on trade conditions). *Assume a fixed distribution* $F(y)$ *for spot prices. Then*

$$\frac{\partial\bar{s}}{\partial m} = 0, \quad \frac{\partial\underline{s}}{\partial m} > 0, \quad \frac{\partial(\bar{s} - \underline{s})}{\partial m} < 0.$$

Proof. This is a direct consequence of Lemma 4. □

Corollary 6 implies that higher market access risk m results in a higher minimum value of $\underline{s}$ and a smaller spread of feasible trade conditions $(\bar{s} - \underline{s})$. As the Seller has better market access, he will have less incentive to participate in the forward contract market, i.e., the minimum value for the reservation fee s for feasible contract trade increases. When he has perfect market access, i.e., $m = 1$, then the feasible set for both parties to trade becomes empty and there is no long-term forward market at all. This same result was noted in Corollary 4.

Finally, we briefly discuss why neither the Seller nor the Buyer would buy on the day from the spot market and sell short in the forward market. To this end, we note that for any firm (either the Seller or the Buyer) to engage in such an activity, the net unit expected profit would be

$$E\{s + g\chi\{P_s > g\} - P_s\} = s + g[1 - F(g)] - \mu$$
$$= s + G(g) - \mu - \int_0^g P_s f(P_s)\,\mathrm{d}P_s < 0.$$

The second equality holds due to (8). The inequality holds due to Lemma 4, since no Buyer would be willing to sign any contract if the index exceeds the expected spot market price. Thus, $s + G(g) \leqslant G(U'(0)) < \mu$ must be satisfied if Buyers are to have an incentive to participate in the forward market. Hence, the net unit expected profit from short sales is strictly negative. Thus, no firm would be interesting in such an activity. We summarize this point in the following Corollary 7.

Corollary 7 (Disincentive for selling short). *Neither the Seller nor the Buyers have incentives to sell short in the forward contract market and cover their positions in the spot market.*

3. Single Seller, multi Buyer

This section extends our previous results to the more general case where there are multiple Buyers purchasing from the same Seller and using a radial transportation network. (Incorporating the cost of transportation is straightforward as it requires changing only the variable cost structure of the model, with no other essential consequences.)

Assume there are J Buyers (or customers) with demand functions $D_j(P_s, g, Q_j),\ j = 1, \ldots, J$. We assume that the Buyers and the Seller exchange bids through an "electronic bulletin board" on which they post bids and offers until they have reached an equilibrium. The offers by the Seller are simply the Seller's reservation charge s, contract execution cost g. Buyers post their demands at the indicated contract offers and the market clears when no further adjustment occurs. We again assume that the Seller is a von Stackelberg leader, anticipating the demands of the Buyers when it posts (s, g). Each Buyer determines its optimal contract $Q_j(s, g)$. Based on our earlier results on the structure of Buyer demand and contracting decisions, we obtain the following Theorem.

Define $Y = \sum_{j=1}^{J} Q_j$, where Q_j is determined using Theorem 1.

Theorem 4 (Optimal bidding with multiple Buyers). *The Seller's optimal bidding policy when facing multiple Buyers is,*

$$s^* = \frac{m(\mu - G(b))}{1 - 1/(\epsilon_s(Y))}, \qquad g^* = b.$$

Proof. Trivial. Similar to (13), a key identity used in the proof is

$$\frac{\partial Y}{\partial g} = [1 - F(g)]\frac{\partial Y}{\partial s}. \qquad \square$$

Practically, what Theorem 4 says is that the optimal bidding policy when the Seller facing multiple Buyers is essentially the same as facing one Buyer, treating all Buyers as one aggregated Buyer, by replacing Q_j in Theorem 2 with aggregated demand $Y = \sum_{j=1}^{J} Q_j$. If Buyers' aggregated demand exceeds the Seller's capacity, then Buyers are offered contracts in order of their WTP. In standard marketing science, this can be done practically using the following method. Given the demand functions $D_{sj}(p),\ p \geqslant 0,\ j = 1, \ldots, J$, define the aggregate retail demand as $D_s(p) = \sum_{j=1}^{J} D_{sj}(p)$ and aggregate marginal WTP as $U'(Q) = D_s^{-1}(Q), Q \geqslant 0$. Moreover, if Buyers are offered contracts in order of their WTP, then total demand for the Seller will be the same as when the Seller faces a single Buyer with demand $D_s(p)$ and marginal WTP $U'(Q)$.

4. Summary

This paper models the strategic interaction of long-term contracting and spot market transactions between one Seller and one or more Buyers. The basic model proposed allows the Seller and Buyers to negotiate bilateral contracts for a non-storable good or service over weekly, monthly or longer periods in advance and then, on the day, to sell or buy the good or service in question in an associated spot market. The model assumes that the good in question is non-storable, as in the case of such markets as electric power, hotels and other service industries with dated/perishable goods. The same model would apply, however, to cases in which Buyers do not use intermediate inventory as a matter of policy, but rely on their Sellers for JIT delivery, as would certainly be the case for expensive capital goods. The Seller attempts to assure reasonable utilization of their capacity through bilateral contracts and Buyers, who face uncertainty in the spot market, attempt to hedge against price volatility through the same bilateral contracts.

This paper has derived the Seller's optimal bidding strategies and Buyers' contracting strategies in a von Stackelberg game theoretical framework with the Seller as the leader. We show that Buyers' optimal reservation level follows an index that combines the Seller's reservation cost and execution cost. The Seller's optimal strategy is to set the execution cost as low as possible, i.e., reveal its production cost, but extract its margin from the Buyers using the reservation charge. The optimal reservation charge depends on the opportunity cost the Seller loses from the spot market as well as the inverse elasticity of the Buyers' contract demand (reservation level). The paper proceeds to solve the single Seller–single Buyer supply chain case first. These results were then generalized to multiple Buyers. We have also generalized many of the above results to the case of multi-Seller, multi-Buyer general network (Part II of this work, see [41,42]).

As a Referee of this paper noted, the results on optimal contracts provided here are as applicable to physical bilateral contracts (i.e., forward contracts with actual physical delivery) as to financial contracts (i.e., options which are marked to the Spot Market). Both forward and options contracts (e.g., so-called Contracts for Differences, see [10]) are familiar in industries such as electric power, where they are increasingly serving as a major vehicle for price discovery and intertemporal coordination of supply and demand.

Besides the extensions noted to the multi-Seller, multi-Buyer case, the results of this paper suggest a number of important directions for continuing this research. On the theoretical side, these results need to be generalized to allow for continuous-time pricing dynamics and to allow for state-dependent WTP functions by Buyers and possibly state-dependent cost functions for the Sellers. Concerning continuous-time results, preliminary results are in [38]. For state-dependent utility, the foundational Lemmas 1 and 2 continue to hold (at the corresponding state-dependent utility functions $U(D(P_s(\omega)), \omega)$), but the conditions for optimal contracting policies are much more complex.

A second extension of interest (as noted by a Referee) is to the case in which not only the Seller faces access risks (in the form of "m") but also Buyers. Clearly, such access risk that might prevent them from filling their demands on the spot market would increase their desire to be active in the forward market.

A third theoretical extension concerns results for closed spot markets in which Sellers or Buyers may have market power and therefore may have some influence on spot market price. Here, we assume that the price in the spot market is not influenced by either the Seller or the Buyers, as would be the case with efficiently functioning markets with many participants. In the same spirit, it would be useful to allow the spot markets accessible to the Seller, for sale of excess capacity, to be different than those available to Buyers, for purchase of additional output. The same logic as that developed here would certainly apply, but clearly differential access conditions would affect both contract demand/supply, as well as demand/supply on the day.

On the empirical side, it will be interesting to explore the increasing use of bilateral contracting in energy and service markets to exploit these results in understanding the structure of multi-tiered markets, especially those in which financial intermediaries play an important role.

Finally, while the above analysis employs a traditional analytical approach, it will be interesting to explore the use of artificial intelligent agents to develop near-optimal contracting and bidding strategies [34–36] in more complex environments (e.g., those involving more complex contracting of service quality features than can be represented in an analytical model). The theoretical results presented here can provide a useful benchmark for this analysis, which would allow the exploration of learning and computational strategies in these more realistic environments.

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Appendix A. Proof of Lemma 1

Sketch of Proof:

1. if $P_s \leqslant g$

$$V(D;Q,P_s) = U(D) - P_s D - sQ,$$

the optimal D is

$$D(P_s) = U'^{-1}(P_s) = D_s(P_s).$$

2. if $P_s > g$, then

$$\begin{aligned} &V(D;Q,P_s)\\ &= U(D) - P_s D + (P_s - g)^+ \min[D,Q] - sQ\\ &= \begin{cases} U(D) - gD - sQ & D \leqslant Q,\\ U(D) - P_s D + (P_s - g)Q - sQ & D > Q, \end{cases}\end{aligned}$$

$$\frac{\partial V}{\partial D} = \begin{cases} U'(D) - g & D \leqslant Q,\\ U'(D) - P_s & D > Q, \end{cases}$$

2.1. if $P_s \geqslant U'(Q) > g$, then $D(P_s) = Q$;
2.2. if $U'(Q) > P_s \geqslant g$, then $D(P_s) = U'^{-1}(P_s) = D_s(P_s)$;
2.3. if $P_s > g \geqslant U'(Q)$, then $D(P_s) = U'^{-1}(g) = D_s(g)$.

Combining the results of these two cases, we obtain

$$\begin{aligned} D(P_s,Q) &= D_s(P_s)(1 - \chi(P_s - g))\\ &\quad + \min[\max[Q, D_s(P_s)], D_s(g)]\chi(P_s - g). \end{aligned} \tag{A.1}$$

Since the Buyer will not contract any amount than needed we can assume, i.e., $Q \leqslant D_s(g) = U'^{-1}(g)$ which implies that $g < U'(Q)$, (A.1) can be simplified as

$$D(P_s,Q) = \begin{cases} D_s(P_s) & P_s < g,\\ \max[Q, D_s(P_s)] & \text{otherwise}, \end{cases}$$

which can be further simplified as

$$D(P_s,Q) = \begin{cases} D_s(P_s) & P_s < U'(Q),\\ Q & \text{otherwise}, \end{cases}$$

or $D(P_s,Q) = \max[D_s(P_s), Q]$, which means that the Buyer's demand is kinked when facing dual sources. Hence the proof. $\square$.

Appendix B. Proof of Theorem 2

From (17), the expected profit of the Seller is

$$\begin{aligned} E\prod(s,g,K) &= [s + (1 - m\chi(g - b))(g - b)(1 - F(g))\\ &\quad - m(\mu - G(\max(g,b)))]Q\\ &\quad + mK(\mu - G(b)) - \beta K. \end{aligned}$$

By Lemma 3, we have $g^* = b$, hence,

$$\begin{aligned} E\prod(s,b,K) &= sQ - mQ(\mu - G(b))\\ &\quad + mK(\mu - G(b)) - \beta K. \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{\partial^2 E\prod(s,b,K)}{\partial s^2} &= -\frac{Q}{s - m(\mu - G(b))}\\ &\quad \times \left(2 + \frac{QU'''(Q)}{U''(Q)}\right)\\ &\quad - \frac{Qf(U'(Q))}{[1 - F(U'(Q))]^2}. \end{aligned}$$

Assume

$$QD_s''(Q) + 2D_s'(Q) \leqslant 0,$$

or equivalently,

$$QU'''(Q) + 2U''(Q) \leqslant 0,$$

then

$$\frac{\partial^2 \prod}{\partial s^2} < 0.$$

Hence, $E\prod(s,b,K)$ is strictly concave in s. Therefore, the solution given by the FOC below is unique and globally optimal.

$$\frac{\partial E\prod(s,b,K)}{\partial s} = Q + [s - m(\mu - G(b))]\frac{\partial Q}{\partial s} = 0,$$

this in combination with Theorem 1 gives,

$$s^* = \begin{cases} \frac{m(\mu - G(b))}{1-1/(\epsilon_s(Q))} = \frac{mE(P_s - b)^+}{1-1/(\epsilon_s(Q))} & Q < K, \\ G(U'(K)) - G(b) & Q \geqslant K, \end{cases}$$

$$g^* = b.$$

Thus, Theorem 2 is proved. □

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