Selling or Leasing? Pricing Information Goods with Depreciation of Consumer Valuation

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Should a monopolistic vendor adopt the selling model or the leasing model for information goods or services? We study this question in the context of consumer valuation depreciation. Using a two-period game-theoretic model, we consider two types of consumer-side valuation depreciation for information goods or services: vintage depreciation and individual depreciation. Vintage depreciation assumes that a good or service loses some of its appeal to consumers as it becomes dated, and this effect persists independent of usage. Individual depreciation instead assumes that valuation depreciation happens only for consumers who have consumed or experienced the good or service. We identify conditions under which each pricing model is preferred. For vintage depreciation information goods, the leasing model dominates the selling model in vendor profit. For individual depreciation information goods, the selling model dominates the leasing model as long as the magnitude of individual depreciation exceeds a certain threshold; otherwise, leasing dominates selling. We consider several model extensions such as when network effects are present. Furthermore, we show a negative interaction effect between vintage depreciation and network effects in vendor profit; in contrast, the interaction effect between individual depreciation and network effects can be either negative or positive depending on the magnitude of individual depreciation. Managerial implications are also discussed.

Key words: pricing strategies; selling; leasing; vintage depreciation; individual depreciation; network effects; information goods or services

1. Introduction

To sell or to lease? Practitioners and academic researchers have been engaged in this debate for decades. With the rapid advancement of information technologies, especially the Internet, vendors of information goods and services are increasingly embracing the leasing model (thereafter "leasing" for short), as exemplified by video streaming services (e.g., Netflix, Apple iTunes), online storage services (e.g., Dropbox.com), and software-as-a-service (e.g., Microsoft Office 365). Rather than taking the perpetual ownership, under the leasing model, users rent information goods or services from the vendor and pay a periodic renting fee. In contrast, under the selling model (thereafter "selling" for short), users pay a lump-sum price for the perpetual usage of information goods or services, such as boxed software (e.g., Microsoft Windows, Adobe, Autodesk, SAP), DVDs, hosted solutions (e.g., Rackspace), and mobile applications (e.g., paid apps in Google Play and Apple App Store). The extant literature documents that leasing is more efficient for extracting consumer surplus *over time*. The pioneering work by Coase (1972) suggests that selling is suboptimal for a monopolistic vendor because consumers expect price markdowns and delay their purchases. Leasing is favored over selling because it can eliminate such strategic waiting behaviors (Bond and Samuelson 1984).

We reinvestigate this tradeoff by incorporating another important issue over the timeline, namely, the depreciation of *consumer valuation*. While the extant literature has focused on the quality decay on the product side, we examine the valuation depreciation on the consumer side. Consumer side depreciation is ubiquitous in markets for information goods, such as books, CDs and DVDs, and video games, etc. They share the common feature that their physical attributes hardly depreciate, but their consumption value to consumers diminishes as consumers become satiated (Ishihara and Ching 2012). In the market for video games, Shiller (2013) reports an empirical evidence that consumers may tire quickly of playing. For the average game, high valuation consumers reduce their valuation from $80 in the first month of use to just a couple of dollars by the sixth month. Consumer valuation depreciation is prevalent among information goods which are often intangible (i.e., with no physical features) and their consumption value is sensitive to consumer's experience over time (Shapiro and Varian 1999). To the best of our knowledge, analytical modeling research on consumer side valuation depreciation is largely missing.

To fill this gap, we employ a two-period game-theoretic model to examine two types of consumer valuation depreciation: *vintage depreciation* and *individual depreciation*. Vintage depreciation assumes that a good or service loses some of its appeal to consumers as it becomes dated and this effect persists independent of usage. For example, a dated version of Microsoft Office software is valued much less than a new release. In contrast, individual depreciation assumes that valuation depreciation occurs only when a consumer has already consumed (or experienced) the good or service (Hu 2005). We reexamine vintage depreciation durable goods but in the context of information goods where we focus on the consumer side (Hu 2005), rather than on the product side studied in the literature of durable physical goods (e.g., Desai and Purohit 1998). We use the case of vintage depreciation as a benchmark for the case of individual depreciation, the main focus of our paper.

In an extension to our baseline model, we also include network effects which is a unique feature of information goods (e.g., Katz and Shapiro 1985, Farrell and Saloner 1986). Many information goods, such as online games and chatting tools, are built on user networks or communities in which the user's willingness to pay (WTP) depends on peer adoptions. Following prior literature (e.g., Conner 1995), we model such network effects by using an additive utility function. This functional form allows us to capture both a standalone utility the user gains from consumption and the additional utility the user obtains by interacting with other users of the information good. For example, in video games, the standalone utility can come from single-player game playing and media streaming. A player can also obtain the additional utility by playing with peers over the network.

We identify optimal conditions for each pricing model. First and as a benchmark, for vintage depreciation information goods, leasing dominates selling. This finding extends the extant literature which has largely focused on durable physical goods with vintage depreciation to our context of information goods. Further, we extend this finding to consider network effects, which is new. Second, for individual depreciation information goods, which has largely been missing in the literature but the main focus of this paper, we show that selling dominates leasing when the magnitude of individual depreciation exceeds a certain threshold; otherwise, leasing dominates selling. These findings are new, and they are also extended in the presence of network effects. Furthermore, we show a negative interaction effect between vintage depreciation and network effects in vendor profit; in contrast, the interaction effect between individual depreciation and network effects can be either negative or positive depending on the magnitude of individual depreciation. Strategic implications of the above findings for practitioners are discussed throughout the paper.

The rest of this paper is organized as follows: Section 2 reviews related literature. In Section 3, we first introduce our model assumptions, and then construct a very simple two-consumer example to illustrate the key ideas and insights of our analytical model. We then establish the baseline case of vintage depreciation in Section 4 and the case of individual depreciation in Section 5. Several model extensions, including network effects, are discussed in Section 6. Section 7 outlines the managerial implications and concludes.

2. Literature Review

The rich academic debate about whether to sell or lease durable goods can be traced back at least to Coase (1972). The key idea, as summarized by Bond and Samuelson (1984), is that a monopoly seller of a durable good is effectively unable to exercise its monopoly power. Once an initial stock of the good has been produced and sold, the monopolist still faces the residual demand. Exploiting the residual demand by selling some additional quantity of the good, presumably at a lower price, allows the firm to earn additional profit. Therefore, the monopoly seller's optimal stock level would converge to the competitive stock level at which the market price is equal to the marginal production cost. Rational consumers, on the other hand, will anticipate the price markdown and accordingly value the good only at the competitive price. As a result, the monopolist can earn no more profit than that of a competitive firm – a prediction known as the Coase conjecture. Coase (1972) also suggests that leasing, rather than selling, can improve vendor profit because leasing limits the market supply to the monopoly level, which helps maintain the monopoly price.

Most analytical models in the literature related to the Coase conjecture use two main approaches. The first directly formalizes the stock-level decision Coase (1972) considers. In particular, the vendor of durable goods chooses the amount of available stock at the beginning of each period (Swan 1970, Bulow 1982, Bond and Samuelson 1984, Gul et al. 1986, Suslow 1986, Bhaskaran and Gilbert 2009)[1]. In the stock-level decision models, the Coase conjecture is equivalent to the notion that the monopoly seller's optimal stock level would converge to the competitive stock level. The (inverse) demand functions in these papers are aggregated measures of consumptions at the market level, rather than at the individual level. Because this traditional approach does not model individual consumer behaviors, it cannot differentiate existing adopters who are subject to individual depreciation from potential new consumers who are not. In sum, the stock-level decision models cannot capture consumer-side valuation depreciation, which is the focus of this paper.

The second approach to examine the Coase conjecture models individual consumer behaviors. In this literature, the optimal prices are obtained from the distribution of consumer utilities. Such a model setting allows analysis of consumers' choices at the individual level. In this stream of research, Stokey (1979) investigates the durable goods pricing problem in a continuous time model with a wide range of utility functions. Similar to Coase's criticism to the selling model, Stokey finds that price discrimination over time is not optimal for the seller due to consumer waiting behaviors. Conlisk et al. (1984) revisit Stokey's utility-based model by considering new consumer arrivals in each period. They find that the selling model may still be optimal with new consumer arrivals in each period. Unfortunately, these two studies do not analytically solve the case of leasing. Bagnoli et al. (1989) discover that selling is better than leasing when the consumer valuation is discrete. The utility-based model is also popular in marketing (e.g., Desai and Purohit 1998, Bhaskaran and Gilbert 2005), industrial organization (e.g., Bensaid 1996), and information systems economics (e.g., Chien and Chu 2008, Zhang and Seidmann 2010). Our work follows this stream, and in our utility-based model, we use a standard two-period setting, similar to Conlisk et al. (1984), Desai and Purohit (1998), and Bhaskaran and Gilbert (2009). We contribute to this literature stream by considering individual depreciation associated with the sale or lease of information goods, which is novel. Additionally, we also offer new insights by extending the literature on product side vintage

[1] In this literature, a few papers address quality decay of physical products, such as Bond and Samuelson (1984) and Suslow (1986). However, as pointed out by Desai and Purohit (1998), they do not differentiate existing adopters from potential new adopters.

depreciation (for physical durable goods such as cars, e.g., Desai and Purohit 1998) to the consumer side, in the context of information goods, with and without network effects.

Our work also relates closely to the burgeoning literature on software pricing. For example, Jain and Kannan (2002) examine the server cost structure and compare pricing schemes of information goods. In an auction setting with demand uncertainty, Bhargava and Sundaresan (2004) show that a pay-as-you-go model is optimal when consumer valuation and demand realization correlate negatively. Huang and Sundararajan (2005) compare on-demand and in-house computing from a cost perspective, and discuss the optimal transition path from in-house to on-demand computing. Choudhary (2007) endogenizes the software upgrading decision and shows that the subscription model is always optimal. In our paper, the leasing (selling) model is a simplified form of the subscription (perpetual licensing) pricing model in the software industry. Therefore, our results can also provide novel insights to vendors of software products that exhibit characteristics of individual depreciation.

3. Model Assumptions and An Illustrative Two-Consumer Example

In this section, we first introduce our model assumptions. We then construct a simple example to illustrate the impact of two types of consumer side valuation depreciation: vintage depreciation and individual depreciation.

3.1. Model Assumptions

A monopolistic vendor offers an information good or service with a lifecycle of two periods. The vendor wishes to maximize his total profit over both periods. We assume the marginal production cost of the information good or service is zero (e.g., Shapiro and Varian 1999). We examine and compare two representative pricing models: selling and leasing. Under selling, the vendor announces p_i at the beginning of period $i \in \{1, 2\}$, and consumers pay p_i for the perpetual ownership and usage of the information good. Under leasing, the vendor announces the periodic leasing fee r_i at the beginning of each period $i \in \{1, 2\}$, and consumers pay r_i for the single-period usage for that particular period i. We follow the literature to assume that the vendor is unable to commit to future price path (e.g., Coase 1972, Bulow 1982, Katz and Shapiro 1985). For simplicity, following the durable goods pricing literature (e.g., Conlisk et al. 1984), we assume that no consumer buys more than one unit, and there are no resales. There are no disposal or switching costs if period 1 adopters stop the adoption in period 2.

v	Consumer type, $v \sim$U$[-K,1]$, where $K=0$ stands for the case of no network effects;
$1-\theta$	Magnitude of depreciation $\theta \in [0,1]$;
v_i	Marginal consumer type in period $i \in \{1,2\}$;
p_i	Selling price in period $i \in \{1,2\}$;
r_i	Leasing fee in period $i \in \{1,2\}$;
N_i	Number of paying (under either selling or leasing) consumers at the equilibrium in period $i \in \{1,2\}$;
$S(L)$	Indicator of the selling (leasing) model;
$A(B)$	Indicator of vintage depreciation (individual depreciation) information goods;
s	Strength of network effects, $s \in [0,1]$;
$\pi(\tilde{\pi})$	The vendor's overall profit without (with) network effects;
$\pi_2(\tilde{\pi}_2)$	The vendor's single-period profit in period 2 without (with) network effects;
$U(\tilde{U})$	Consumer utility without (with) network effects;
Ω	Consumer's adoption status over two periods, $\Omega \in \{DD, OD, DO, OO\}$, where D stands for "adopting" and O for "not adopting".

Table 1 Summary of Key Notation

We assume a unit mass of heterogeneous consumers with their type v uniformly distributed on $[-K,1]$ where $K \geq 0$. Therefore the density of consumer distribution is $\frac{1}{1+K}$ everywhere. Specifically, for consumers with $v \in [0,1]$, their type v represents period 1 valuation of the information good, which is subject to depreciation in period 2. In contrast, consumers distributed on $[-K,0)$ are "not interested" in the information good or service. Thus their valuation is equal to 0 in both periods.

In our baseline model without network effects, we only need to consider consumers with $v \in [0,1]$. In Section 6.1, we will extend this baseline model to incorporate network effects, under which those consumers with type $v < 0$ may become "interested" if they expect benefits generated by network effects (e.g., Conner 1995, Jing 2007).

Next we introduce the mechanisms of valuation depreciation specifically for consumers with nonnegative valuation. For *vintage depreciation* information goods, any consumer v's valuation will depreciate from v to θv in period 2; in contrast, for *individual depreciation* information goods, consumer v's valuation will depreciate to θv only when she is a period 1 adopter. Otherwise, consumer v will maintain the same initial valuation in period 2 (Hu 2005). We denote N_i as the number of paying consumers (either under leasing or selling) at the equilibrium in period i. We summarize our key notation in Table 1. For both vintage depreciation information goods and individual depreciation information goods, we compare and contrast the selling model with the leasing model.

3.2. An Illustrative Two-Consumer Example

Consider a market with two consumers, denoted by V_1 and V_2, with an initial single-period valuation of 10 and 4 at the beginning of period 1, respectively. The information good has a two-period lifecycle. In period 2, consumer valuation will drop by 75% (i.e., $\theta = 0.25$) when depreciation applies.

In the scenario of vintage depreciation, both consumers' valuation is subject to depreciation in period 2, unconditionally. Thus consumers' period 2 valuation is $10 \times 0.25 = 2.5$ and $4 \times 0.25 = 1$, respectively.

Under selling and vintage depreciation, if consumer V_1 purchases in period 1, then only consumer V_2 is left in period 2 and optimal period 2 selling price is 1. While consumer V_1 has an overall valuation of $10 + 2.5 = 12.5$ over two periods, the vendor is unable to charge a price at 12.5. Why? Because consumer V_1 would delay her adoption to period 2 when the selling price is 1 so as to enjoy a greater surplus of $2.5 - 1 = 1.5$, rather than purchasing in period 1 with a zero surplus $(12.5 - 12.5 = 0)$. Thus, the incentive compatibility constraint leads to the optimal period 1 price at $12.5 - 1.5 = 11$. The total profit for a selling vendor is then $11 + 1 = 12$.

Under leasing and vintage depreciation, in period 2, the optimal leasing fee is 2.5, and period 2 profit is 2.5 because only consumer V_1 rents. In period 1, the optimal leasing fee is 10 and again only consumer V_1 rents. The total profit for a leasing vendor is $2.5 + 10 = 12.5$. Thus under vintage depreciation, leasing dominates selling in vendor profit $(12.5 > 12)$.

In the scenario of individual depreciation, only the valuation of period 1 adopter depreciates. For example, if we assume that only consumer V_1 adopts in period 1, then her valuation drops to $10 \times 0.25 = 2.5$ in period 2, while consumer V_2 maintains the period 1 valuation of 4. Intuitively, one would expect that the vendor's profit should be at least equal to that under vintage depreciation because depreciation only occurs to existing adopters, rather than to all consumers. However, we show that the vendor becomes worse off in this scenario.

Under selling and individual depreciation, if consumer V_1 adopts in period 1, then only consumer V_2 is left at the beginning of period 2. The optimal selling price in period 2 is 4. Given this, the optimal selling price in period 1 is 6.5 after considering the incentive compatibility constraint. Otherwise, consumer V_1 would delay her adoption until period 2 for a greater surplus. The total profit for a selling vendor is 10.5.

Under leasing and individual depreciation, in period 2, if we assume that only consumer V_1 adopts in period 1, then her valuation drops to 2.5 in period 2, while consumer V_2 maintains the valuation of 4. Can the vendor increase period 2 leasing fee to 4? Unfortunately not, because consumer V_1 would then delay her adoption to period 2 and receive a surplus of $10 - 4 = 6$, instead of adopting early in period 1 (in which case her surplus is $0 < 6$). As a result, the optimal period 2 leasing fee is 10. No consumers rent in period 1, and only consumer V_1 rents in period 2. The total profit for a leasing vendor is 10. Thus under individual depreciation, leasing can make less profit than selling (i.e., $10 < 10.5$).

Table 2 summarizes the vendor's profits under our 2×2 design between depreciation types and pricing models. Our simple two-consumer example illustrates the following key insights of our paper: Benchmarked with vintage depreciation, in the presence of individual depreciation, consumers have incentives to wait, both under selling and under leasing. This, in turn, hurts the vendor's profit. Surprisingly, selling can mitigate such customer's waiting behavior better than

	Selling	Leasing
Vintage Depreciation	12	12.5
Individual Depreciation	10.5	10

Table 2 Summary of Profits in Two-Consumer Example

leasing can, particularly, when the magnitude of individual depreciation is large. We formalize these insights in our analytical models below.

4. Vintage Depreciation

As a benchmark, we start our analysis by studying vintage depreciation information goods, where all consumers, including those who do not adopt in period 1, depreciate their valuation in period 2. As a result, in period 2, consumer valuation is uniformly distributed on $[0,\theta]$ (with a greater density). The update to the distribution of consumer valuation, exemplified with $\theta = 0.5$, is illustrated in Figure 1.

Figure 1 The distribution density function of consumers' single-period valuation ($\theta = 0.5, K = 0$)

4.1. The Selling Model

Under selling, the vendor announces p_i at the beginning of period $i \in \{1,2\}$. Denote the marginal consumer type in period i as v_i ($i \in \{1,2\}$). This marginal consumer type represents consumers who are indifferent between adopting and not adopting in each period. We solve the vendor's problem using backward induction.

At the beginning of period 2, the vendor only needs to consider potential consumers with type distributed on $[0, v_1]$, because period 1 adopters have purchased the information good with perpetual access to use. For any consumer $v \in [0, v_1]$, period 2 valuation depreciates to θv. She will become a period 2 adopter if p_2 satisfies $\theta v \geq p_2$. The population of paying consumers in period 2 is given by $N_2 = v_1 - v_2$, in which v_2 can be determined by solving $\theta v_2 - p_2 = 0$. The vendor's period 2 problem is

$$\max_{v_2 \in [0, v_1]} \quad \pi_2^{S,A}(v_2|v_1) = (v_1 - v_2)\theta v_2,$$

where the superscripts S and A stand for selling and vintage depreciation information goods, respectively. Solving, we have $v_2^* = \frac{v_1}{2}$, $p_2^* = \frac{\theta v_1}{2}$, and the vendor's optimal period 2 profit is $(\pi_2^{S,A})^* = \frac{\theta v_1^2}{4}$.

Next, we move to period 1. A type-v consumer's utility function is denoted by $U_v(\Omega)$, where $\Omega \in \{DD, OD, DO, OO\}$ stands for consumer v's adoption status in each period (D for "adopting" and O for "not adopting"). In period 1, all consumers have three options: buying in period 1 (i.e., adopting in both periods, denoted by $\Omega = DD$), delaying adoption to period 2 (i.e., $\Omega = OD$), or never adopting (i.e., $\Omega = OO$). The corresponding utility function $U_v(\Omega)$ is in Equation (1).

$$\begin{aligned} U_v^{S,A}(DD) &= (1+\theta)v - p_1; \\ U_v^{S,A}(OD) &= \theta v - p_2; \\ U_v^{S,A}(OO) &= 0. \end{aligned} \tag{1}$$

Following the literature (e.g., Fudenberg and Tirole 1991), we assume that, at the beginning of period 1, consumers can correctly expect the vendor's period 2 optimal pricing strategy at the rational expectations equilibrium (REE). Marginal consumers (with type v_1) are indifferent between adopting in period 1 and delaying adoption until period 2. Therefore, v_1 can be obtained by solving $U_{v_1}^{S,A}(DD) = U_{v_1}^{S,A}(OD)$, which yields, $p_1 = \left(1 + \frac{\theta}{2}\right) v_1$. The number of period 1 paying consumers is $N_1 = 1 - v_1$. The vendor's period 1 problem is

$$\begin{aligned} \max_{v_1 \in [0,1]} \quad & \pi^{S,A}(v_1) = p_1 N_1 + (\pi_2^{S,A})^*(v_1), \\ s.t. \quad & p_1 = \left(1 + \frac{\theta}{2}\right) v_1, \\ & N_1 = 1 - v_1. \end{aligned}$$

Solving, we have

$$p_1^* = \frac{(2+\theta)^2}{2(4+\theta)}, \ p_2^* = \frac{\theta(2+\theta)}{2(4+\theta)}, \ (\pi^{S,A})^* = \frac{(2+\theta)^2}{4(4+\theta)}. \tag{2}$$

The numbers of paying consumers are

$$N_1^* = \frac{2}{4+\theta}, \ N_2^* = \frac{2+\theta}{2(4+\theta)}. \tag{3}$$

Figure 2 **Consumers' valuation and adoptions under selling and vintage depreciation ($\theta = 0.5, K = 0$)**

We depict consumer adoptions under the optimal selling strategy in Figure 2. Consumers' valuation of "buy in period 1" is always higher than that of "buy in period 2", because the total benefit of buying in period 1 is $(1+\theta)v$, which is always greater than the benefit of buying in period 2, θv. In period 2, a rational vendor lowers the price ($p_2^* < p_1^*$) to induce more purchases. Expecting this future price markdown, a group of consumers (denoted by region W) choose to delay their adoptions until period 2 even if they can afford p_1 (i.e., $(1+\theta)v \geq p_1$). The vendor, in turn, has to lower period 1 price (i.e., $p_1^* < (1+\theta)v_1$) to alleviate such waiting behavior. If consumers do not wait, i.e., they adopt in period 1 as long as $U^{S,A}(DD) \geq 0$, then the vendor's optimal profit is $\frac{(1+\theta)^2}{4+3\theta}$. We denote the profit loss incurred due to such consumer waiting by $Loss^{S,A}$. Then, we have

$$Loss^{S,A} = \frac{(1+\theta)^2}{4+3\theta} - (\pi^{S,A})^* = \frac{(1+\theta)^2}{4+3\theta} - \frac{(2+\theta)^2}{4(4+\theta)} = \frac{\theta[8+\theta(8+\theta)]}{4(4+\theta)(4+3\theta)}.$$

4.2. The Leasing Model

Under leasing, the vendor announces the leasing fee r_i at the beginning of each period $i \in \{1, 2\}$. All consumers, including existing adopters in period 1, must pay the leasing fee in period 2 if they opt to use the information good. Again we solve the vendor's problem using backward induction.

Consider period 2 first. Unlike a selling vendor, a leasing vendor needs to take all consumers into consideration. The number of paying consumers in period 2 is $N_2 = 1 - v_2$ where v_2 satisfies $\theta v_2 - r_2 = 0$. The vendor's period 2 profit is $\pi_2^{L,A}(v_2) = N_2 \times r_2 = (1-v_2)\theta v_2$. The superscript L represents the leasing model. Solving, we have $v_2^* = \frac{1}{2}$, $r_2^* = \frac{\theta}{2}$, and $(\pi_2^{L,A})^* = \frac{\theta}{4}$.

At the beginning of period 1, in contrast to selling, leasing consumers have the freedom to rent only in period 1. Therefore, there are four candidate strategies:

$$\begin{aligned}
U_v^{L,A}(DD) &= (1+\theta)v - r_1 - r_2;\\
U_v^{L,A}(DO) &= v - r_1;\\
U_v^{L,A}(OD) &= \theta v - r_2;\\
U_v^{L,A}(OO) &= 0.
\end{aligned}$$

Under REE, the marginal consumer type v_1 satisfies either $U_{v_1}^{L,A}(DD) = U_{v_1}^{L,A}(OD)$ (when there are new adopters in period 2) or $U_{v_1}^{L,A}(DO) = 0$ (when some consumers only rent in period 1), both resulting $r_1 = v_1$ after simplification. Thus, the vendor's problem becomes

$$\begin{aligned} \max_{v_1 \in [0,1]} \quad & \pi^{L,A}(v_1) = r_1 N_1 + (\pi_2^{L,A})^*(v_1), \\ s.t. \quad & r_1 = v_1, \\ & N_1 = 1 - v_1. \end{aligned}$$

The optimal solution is $v_1^* = \frac{1}{2}$ which gives $r_1^* = \frac{1}{2}$ and $(\pi^{L,A})^* = \frac{1+\theta}{4}$. The numbers of paying consumers in each period are $N_1^* = N_2^* = \frac{1}{2}$. Figure 3 illustrates consumers' valuation and adoption under leasing and vintage depreciation.

Figure 3 Consumers' valuation and adoptions under leasing and vintage depreciation ($\theta = 0.5, K = 0$)

In each period, the marginal consumers' adoption decisions only depend on single-period valuation and the leasing fee (i.e., $v_1 = r_1$, $\theta v_2 = r_2$). That is, consumers do not wait under leasing because they do not take the future price into consideration, i.e., $Loss^{L,A} = 0$. Thus leasing effectively eliminates consumer waiting behavior. Comparing the vendor's profit under leasing and selling, we have

$$(\pi^{L,A})^* - (\pi^{S,A})^* = \frac{1+\theta}{4} - \frac{(2+\theta)^2}{4(4+\theta)} = \frac{\theta}{4(4+\theta)} \geq 0,$$

which leads to the following Proposition 1.

Proposition 1. *For vintage depreciation information goods, leasing dominates selling in vendor profit.*

Note that

$$\frac{Loss^{S,A}}{(\pi^{L,A})^* - (\pi^{S,A})^*} = \frac{8+\theta(8+\theta)}{4+3\theta} \geq 1, \text{ for } \theta \in [0,1],$$

which implies that selling would dominate leasing if consumers did not wait. Proposition 1 formalizes the idea in Coase (1972) that leasing can effectively eliminate consumer waiting, but extends it to the context of vintage depreciation information goods.

Interestingly, compared to leasing, selling offers higher social welfare. This can be shown by computing the area below the valuation curves in Figure 2 and 3 among those adopting consumers. We have the following Proposition 2.

Proposition 2. *For vintage depreciation information goods, selling dominates leasing in social welfare.*

Intuitively, selling covers a larger market than leasing, leading to a higher social welfare. However, leasing dominates selling in vendor profit by focusing on the higher value consumer segment. Taken together, these findings extend the extant literature of vintage depreciation durable goods to the new context of durable information goods. In particular, we focus on the consumer-side valuation depreciation, rather than on the product side (e.g., quality decay or physical wear and tear over time, see Desai and Purohit 1998) in the prior literature.

Proposition 1 above provides a useful benchmark for the case of individual depreciation information goods, which is our key focus. It is a new type of consumer valuation depreciation in practice, yet it has not been treated formally in the academic literature. We analyze this case in the next section.

5. Individual Depreciation

For an individual depreciation information good, in period 2, only period 1 adopters depreciate their valuations but not others. As a comparison to Figure 1, we depict the density function of consumer valuation distribution in period 2 in Figure 4. In sharp contrast with Figure 1, the consumer type distribution is no longer uniform in period 2. Specifically, consumers with type $v \in [v_1, 1]$ (i.e., period 1 adopters) depreciate their valuation, while consumers with type $[0, v_1)$ do not. This leads to two cases: (a) $v_1 < \theta$ (see Figure 4a), and (b) $v_1 \geq \theta$ (see Figure 4b). Under case (a), the distribution of period 2 valuation is further segmented into three intervals: (a.1) the interval $[0, \theta v_1)$, which consists of only period 1 non-adopters with a density of 1; (a.2) the interval $[\theta v_1, v_1)$, which consists of both period 1 adopters and non-adopters, with a density of $1 + \frac{1}{\theta}$; and (a.3) the interval $[v_1, \theta]$, which consists of only period 1 adopters with a density of $\frac{1}{\theta}$. Similarly, in case (b), the distribution of period 2 valuation is segmented into three intervals: (b.1) intervals $[0, \theta v_1)$ and $[\theta, v_1]$, which consist of only period 1 non-adopters with a density of 1; and (b.2) the interval $[\theta v_1, \theta)$, which consists of both period 1 adopters and non-adopters with a density of $1 + \frac{1}{\theta}$.

Figure 4 **The distribution density functions of consumers' single-period valuation ($\theta = 0.5, K = 0$)**

Note that, although θ is exogenous, v_1 is determined by the vendor's pricing strategy, which implies that the distribution density of consumers' period 2 valuation can be manipulated by the vendor's period 1 pricing strategy. This insight connects our work with the literature on endogenous demand functions (e.g., Johnson and Myatt 2003, 2006, Bhargava and Chen 2012). For individual depreciation information goods, upon observing θ, the vendor can strategically manipulate consumer valuation towards a certain distribution in period 2. For example, if the vendor charges a smaller period 1 leasing fee to induce more adoptions in period 1 (which leads to a smaller v_1), then period 2 distribution converges to a pattern, as illustrated in Figure 4a, where consumers are segmented into three intervals with different densities. If in period 1, the vendor maintains a relatively small number of adopters by charging a higher period 1 leasing fee, then period 2 distribution converges to a pattern as illustrated in Figure 4b, with a single bump in the center.

5.1. The Selling Model

Under selling, we denote the selling prices as p_i $(i \in \{1, 2\})$ for period i, and solve the vendor's problem using backward induction. At the beginning of period 2, the selling vendor only needs to consider consumers in the interval $[0, v_1]$. The number of paying consumers in period 2 is $N_2 = v_1 - v_2$ where v_2 satisfies $v_2 - p_2 = 0$. Note that v_2 is not affected by individual depreciation, because new adopters in period 2 have not adopted in period 1. The vendor's period 2 problem is

$$\max_{v_2 \in [0, v_1]} \quad \pi_2^{S,B}(v_2 | v_1) = (v_1 - v_2) \times v_2,$$

where the superscript B represents the case of individual depreciation information goods. Solving, we have $v_2^* = \frac{v_1}{2}$, $p_2^* = \frac{v_1}{2}$, and $(\pi_2^{S,B})^* = \frac{v_1^2}{4}$. At the beginning of period 1, consumer v faces three options with following utility function:

$$\begin{aligned} U_v^{S,B}(DD) &= (1 + \theta)v - p_1; \\ U_v^{S,B}(OD) &= v - p_2; \\ U_v^{S,B}(OO) &= 0. \end{aligned}$$

Note that existing adopters need not to pay p_2 for their period 2 adoptions. The marginal consumer type v_1 can be obtained by solving $U_{v_1}^{S,B}(DD) = U_{v_1}^{S,B}(OD)$, which yields, $v_1 = \frac{p_1 - p_2^*}{\theta}$ or equivalently, $p_1 = \theta v_1 + p_2^* = (\theta + \frac{1}{2})v_1$. The number of period 1 paying consumers is $N_1 = 1 - v_1$. Therefore, the vendor's problem is

$$\begin{aligned} \max_{v_1 \in [0,1]} \quad & \pi^{S,B}(v_1) = p_1 N_1 + (\pi_2^{S,B})^*(v_1), \\ s.t. \quad & p_1 = (\theta + \frac{1}{2})v_1, \\ & N_1 = 1 - v_1. \end{aligned}$$

Solving the vendor's problem, we obtain Lemma 1.

Lemma 1. *For individual depreciation information goods and under the selling model, the optimal price strategies (p_1^*, p_2^*) are*

$$p_1^* = \frac{(1+2\theta)^2}{2(1+4\theta)}, \; p_2^* = \frac{1+2\theta}{2(1+4\theta)}.$$

The optimal profit is

$$(\pi^{S,B})^* = \frac{(1+2\theta)^2}{4(1+4\theta)}.$$

The numbers of paying consumers in each period are

$$N_1^* = \frac{2\theta}{(1+4\theta)}, \; N_2^* = \frac{1+2\theta}{2(1+4\theta)}.$$

We illustrate consumer valuation and their adoptions in Figure 5. The parameter setting is the same as in Figure 2. Note that there exists a fraction of strategic consumers (denoted by W) who would delay adoption to period 2. It can be shown that the selling profit without consumer waiting is $\frac{(1+\theta)^2}{3+4\theta}$ (i.e., assuming consumer v adopts in period 1 as long as $U_v^{S,B}(DD) > 0$), thus the vendor's profit loss due to consumer waiting is, $Loss^{S,B} = \frac{(1+\theta)^2}{3+4\theta} - (\pi^{S,B})^* = \frac{1+8\theta+8\theta^2}{4(3+16\theta+16\theta^2)}$. Note that this loss is even greater than that under vintage depreciation, because

$$\frac{Loss^{S,B}}{Loss^{S,A}} \geq 1, \text{ for all } \theta \in [0,1].$$

Thus, consumers have stronger incentives to wait until period 2, in the case of individual depreciation than in the case of vintage depreciation.

Figure 5 **Consumers' valuation and adoptions under selling and individual depreciation ($\theta = 0.5, K = 0$)**

5.2. The Leasing Model

Finally, we examine the leasing model under individual depreciation. The vendor announces a single-period leasing fee r_i at the beginning of each period $i = \{1, 2\}$. We use v_i to denote the marginal consumer type in period i ($i \in \{1, 2\}$). As illustrated in Figure 4, there are two cases to consider here: (a) $v_1 < \theta$ and (b) $v_1 \geq \theta$. The distribution of consumer valuation in period 2 is no longer a single uniform continuum but features intervals of different densities (see Figure 4). Consequently, the vendor's period 2 problem becomes non-trivial, because the marginal consumer type v_2 can be located in any interval, as illustrated in Figure 4.

For case (a) in Figure 4, period 2 paying consumers N_2 depends on the location of marginal consumer type v_2:

$$N_2(r_2|v_1 < \theta) = \begin{cases} 1 - r_2, & r_2 \in [0, \theta v_1); \\ v_1 - r_2 + 1 - \frac{r_2}{\theta}, & r_2 \in [\theta v_1, v_1); \\ 1 - \frac{r_2}{\theta}, & r_2 \in [v_1, \theta]. \end{cases} \tag{4}$$

Similarly, for case (b) in Figure 4, N_2 is given as

$$N_2(r_2|v_1 \geq \theta) = \begin{cases} 1 - r_2, & r_2 \in [0, \theta v_1); \\ v_1 - r_2 + 1 - \frac{r_2}{\theta}, & r_2 \in [\theta v_1, \theta); \\ v_1 - r_2, & r_2 \in [\theta, v_1]. \end{cases} \tag{5}$$

The vendor's period 2 problem is

$$\max_{v_2 \in [0,1]} \ \pi_2^{L,B}(v_2|v_1) = r_2 N_2(r_2|v_1),$$
$$s.t. \ \ r_2 = \begin{cases} \theta v_2, \ v_2 \geq v_1; \\ v_2, \ \ v_2 < v_1. \end{cases}$$

At the beginning of period 1, the type-v consumer's utility function is

$$U_v^{L,B}(DD) = (1 + \theta)v - r_1 - r_2;$$

$$U_v^{L,B}(DO) = v - r_1;$$
$$U_v^{L,B}(OD) = v - r_2;$$
$$U_v^{L,B}(OO) = 0.$$

The vendor's period 1 problem is:

$$\max_{v_1 \in [0,1]} \quad \pi^{L,B}(v_1) = r_1 N_1 + (\pi_2^{L,B})^*(v_1),$$
$$s.t. \quad r_1 = \begin{cases} \theta v_1, \; v_1 \geq v_2^*; \\ v_1, \quad v_1 < v_2^*, \end{cases}$$
$$N_1 = 1 - v_1.$$

Solving, we have Lemma 2.

Lemma 2. *For individual depreciation information goods and under the leasing model, the optimal leasing fees* (r_1^*, r_2^*) *are*

$$(r_1^*, r_2^*) = \begin{cases} (\frac{1}{2} + \epsilon, \frac{1}{2}), \;\; \theta \in [0, \frac{1}{3}); \\ (\frac{\theta}{1+\theta}, \frac{\theta}{1+\theta}), \; \theta \in [\frac{1}{3}, 1]. \end{cases} \tag{6}$$

The optimal profit $(\pi^{L,B})^*$ *is*

$$(\pi^{L,B})^* = \begin{cases} \frac{1}{4}, \quad\;\; \theta \in [0, \frac{1}{3}); \\ \frac{\theta}{1+\theta}, \; \theta \in [\frac{1}{3}, 1]. \end{cases} \tag{7}$$

The numbers of paying consumers in each period (N_1^*, N_2^*) *are*

$$(N_1^*, N_2^*) = \begin{cases} (0, \frac{1}{2}), \qquad\;\; \theta \in [0, \frac{1}{3}); \\ (\frac{\theta}{1+\theta}, \frac{1}{1+\theta}), \; \theta \in [\frac{1}{3}, 1]. \end{cases} \tag{8}$$

Figure 6 Consumers' valuation and adoptions under leasing and individual depreciation ($\theta = 0.5$, $K = 0$)

Lemma 2 states that, when individuals heavily depreciate the valuation, i.e., when $\theta < \frac{1}{3}$, at the equilibrium (REE), consumers opt to wait until period 2 even if they can afford leasing in period 1. For example, suppose $\epsilon = \frac{1}{4}$. Then consumers with type $v \in [\frac{3}{4}, 1]$ would still choose to delay adoption even if they can afford period 1 leasing fee. Otherwise when $\theta \geq \frac{1}{3}$, it is optimal to charge a fixed fee, i.e., the same rental fee at each period ($r_1^* = r_2^*$). Proposition 3 follows immediately from Lemma 1 and Lemma 2.

Proposition 3. *For individual depreciation information goods, selling dominates leasing in vendor profit when $\theta \in [0, \frac{1}{2})$, otherwise when $\theta \in [\frac{1}{2}, 1]$ leasing dominates selling.*

The insight from Proposition 3 is that, for information goods with individual depreciation, selling dominates leasing when the magnitude (i.e., $1-\theta$) of individual depreciation is large. The driving factor behind this is also consumers' waiting behaviors. At the equilibrium (REE), we show that consumers might as well choose to wait under leasing (see region W in Figure 6). If consumers adopt as long as $U_v^{L,B}(DO) \geq 0$, then the leasing vendor's optimal profit is $\frac{1}{3}$ for $\theta \in [0, \frac{1}{6})$, $1 - \frac{3}{4+3\theta}$ for $\theta \in [\frac{1}{6}, \frac{1}{3})$, and $\frac{(1+\theta)^2}{4(1+\theta^2)}$ for $\theta \in [\frac{1}{3}, 1]$. Thus, we can show that, the leasing vendor's profit loss due to such consumer waiting behavior, $Loss^{L,B}$, satisfies that

$$\frac{Loss^{L,B}}{Loss^{S,B}} \geq 1, \text{ for } \theta \in [\underline{\theta}, \frac{1}{2}], \tag{9}$$

where $\underline{\theta}$ is the unique root to the equation $2\underline{\theta}[\underline{\theta}(11+15\underline{\theta})-1] = 1$ on the interval $[\frac{1}{6}, \frac{1}{3})$ ($\theta \approx 0.23$). Equation (9) implies that the loss due to consumer waiting under leasing is larger than that under selling when individual depreciation is large. In this case, selling is favored over leasing because selling can better mitigate consumer waiting behavior.

Interestingly, as we show in Proposition 4 below, selling also provides higher social welfare than leasing in the context of individual depreciation information goods.

Proposition 4. *For individual depreciation information goods, selling dominates leasing in social welfare.*

We next consider several model extensions, and compare them with our baseline model.

6. Extensions

We extend our baseline model along three dimensions: Section 6.1 incorporates network effects; Section 6.2 considers conditional pricing whereby the vendor commits to having a discount in period 2 for period 1 adopters; and Section 6.3 discusses the hybrid pricing model in our two-period setting when the vendor is able to offer both selling and leasing simultaneously.

6.1. Network Effects

Network effects are ubiquitous in markets for information goods (e.g., Katz and Shapiro 1986, 1992, Shapiro and Varian 1999). A natural extension of our baseline model therefore is to examine optimal selling and leasing strategies under network effects. For example, consumers of video games can either play the single-player game alone, or join a game with friends on the player network. This also applies to many information goods that share the feature of online communities, where adopters may benefit from collaborative adoptions with peers. Examples include cloud storage services (e.g., Dropbox) and product discussion forums (e.g., Microsoft community).

Following the literature on network effects, we consider a group of potential consumers with their type v distributed on $v \in [-K, 0)$, where K is large enough such that the vendor can never fully cover the entire market. We denote the utility functions with network effects by $\tilde{U}$. In the presence of network effects, it may become possible that $\tilde{U}_v > 0$, even when $v < 0$, in which case the consumer's adoption is driven solely by peer adoptions. This setup is standard in the literature (e.g., Katz and Shapiro 1985, Conner 1995, Jing 2007). For vintage depreciation information goods studied in section 4.1, we rewrite Equation (1) as the following:

$$\begin{aligned} \tilde{U}_v^{S,A}(DD) &= v + sN_1 + \theta\,[v + s(N_1+N_2)]^+ - p_1; \\ \tilde{U}_v^{S,A}(OD) &= \theta\,[v + s(N_1+N_2)]^+ - p_2; \\ \tilde{U}_v^{S,A}(OO) &= 0, \end{aligned} \tag{10}$$

where s denotes the strength of network effects, and $[v + s(N_1+N_2)]^+ = \max\{v + s(N_1+N_2), 0\}$. The interpretation of this formulation is that the depreciation only occurs when consumers have nonnegative utility. The utility function in Equation (10) generalizes that in Equation (1), as it is straightforward to see that they are identical when $s = 0$. We assume $s \in [0, 1]$ to maintain a reasonable strength of network effects.[2]

[2] This assumption ensures that network effects will not be so strong that it dominates all other factors. We thank the review team for this suggestion.

Proposition 5. *For vintage depreciation information goods with network effects, leasing dominates selling in vendor profit.*

According to Proposition 5, the presence of network effects does not change the dominance of leasing over selling for vintage depreciation information goods. The vendor's profit under both selling and leasing models is influenced by network effects, but in different ways. Under selling, as the strength of network effects increases, consumers have smaller incentives to delay adoption until period 2, because waiting becomes less attractive. Under leasing and in contrast, the optimal leasing fees r_1 and r_2 are not influenced by network effects, but the number of adopters increases as the strength of network effects increases.

In addition to contributing to the literature on vintage depreciation by considering the consumer side valuation depreciation over time, our extensions to consider network effects appear new in the literature. We are able to obtain closed-form solutions for both the selling model and the leasing model in the presence of network effects. This in turn allows us to explicitly compare selling vs. leasing, which is largely missing in the literature.[3] Next we examine the case of individual depreciation with network effects, and Proposition 6 summarizes our results.

Proposition 6. *For individual depreciation information goods with network effects, in terms of vendor profit,*

(i) when $\theta \in [0, \frac{s}{2+2K-s}]$*, selling and leasing are equivalent;*

(ii) when $\theta \in (\frac{s}{2+2K-s}, \frac{1+K-s}{2+2K-s}]$*, selling dominates leasing; and*

(iii) when $\theta \in (\frac{1+K-s}{2+2K-s}, 1]$*, leasing dominates selling.*

We illustrate Proposition 6 in Figure 7. Consistent with our analysis in Section 5.2, leasing induces waiting under individual depreciation. Consequently, when θ is relatively small (i.e., the magnitude of individual depreciation, $1-\theta$, is large), selling dominates leasing. Note also that when network effects do not exist, that is, when $s=0$, Proposition 6 reduces to Proposition 3.

[3] For example, Desai and Purohit (1998) do not study network effects, as they focus on product side vintage depreciation for physical goods rather than consumer side vintage depreciation for information goods as we do. While Bensaid and Lesne (1996) and Mason (2000) consider network effects, neither studies leasing, therefore, their papers are unable to compare selling vs. leasing, one key focus of this paper.

Figure 7 Optimal pricing strategies under individual depreciation with network effects ($K = 7$)

Comparing Proposition 6 to Proposition 5 thus offers insights for pricing information goods with depreciation and network effects. For vintage depreciation information goods, leasing dominates selling. In contrast, the optimal pricing scheme for individual depreciation information goods largely depends on the magnitude of the depreciation, and selling dominates leasing when the magnitude of the depreciation is large.

It is also interesting to note the region in Figure 7 where selling and leasing are equivalent in generating vendor profit. This is so because in this region, individual depreciation is so strong that, at the equilibrium (REE), all consumers would wait (under either leasing or selling) until period 2, making the selling model and the leasing model identical in our two-period setting. Consequently, they generate the same vendor profit.

Next we examine the interaction effects between the strength of network effects and the magnitude of valuation depreciation in vendor profit. We obtain the following results.

Proposition 7. *For vintage depreciation information goods with network effects,*

(i) the vendor's profit is increasing and convex in the strength of network effects (i.e., $\frac{\partial \tilde{\pi}^*}{\partial s} > 0$ *and* $\frac{\partial^2 \tilde{\pi}^*}{\partial s^2} > 0$*);*

(ii) the interaction effect between vintage depreciation and the strength of network effects in vendor profit is negative (i.e., $\frac{\partial^2 \tilde{\pi}^*}{\partial(1-\theta)\partial s} < 0$*).*

Proposition 8. *For individual depreciation information goods with network effects,*

(i) the vendor's profit is increasing and convex in the strength of network effects (i.e., $\frac{\partial \tilde{\pi}^*}{\partial s} > 0$ *and* $\frac{\partial^2 \tilde{\pi}^*}{\partial s^2} > 0$*);*

(ii) there exists a threshold $\hat{\theta} \in (\frac{s}{2+2K-s}, \frac{1+K-s}{2+2K-s}]$*, when* $\theta \in [0, \hat{\theta}]$*, the interaction effect between individual depreciation and the strength of network effects is positive (i.e.,* $\frac{\partial^2 \tilde{\pi}^*}{\partial(1-\theta)\partial s} \geq 0$*);*

(iii) when $\theta \in (\hat{\theta}, 1]$*, the interaction effect between individual depreciation and the strength of network effects is negative (i.e.,* $\frac{\partial^2 \tilde{\pi}^*}{\partial(1-\theta)\partial s} < 0$*).*

While it is not surprising to find that the vendor's marginal profit increases in the strength of network effects under both vintage depreciation and individual depreciation, we note several

Figure 8 Interaction effects between individual valuation depreciation and network effects in vendor profit ($K = 7$)

new and interesting findings. First, when leasing dominates selling (under both types of consumer valuation depreciation), the vendor's profit loss due to stronger valuation depreciation can be remedied by network effects. Theoretically, it means that in the case when leasing dominates selling, network effects mitigates consumer valuation depreciation. Practically, this suggests that stronger network effects not only help a leasing vendor attract more consumers, but also help him alleviate consumer valuation depreciation. Second, we identify a region under which a vendor of individual depreciation information goods favors the selling model over the leasing model. Here we uncover non-trivial dynamics between the interplay of individual valuation depreciation and network effects. The interaction effects between the two factors can be negative or positive, depending on a threshold $\hat{\theta}$ (defined in the appendix). As we illustrated in Figure 8, as the strength of network effects increases from $s = 0$ to $s = 0.8$, when $\theta > \hat{\theta}$, the interaction effect between individual depreciation and network effects is negative; however, when $\theta \leq \hat{\theta}$, the interaction effect becomes positive. In this region, as we show in Proposition 6, the vendor favors selling over leasing. Here both network effects and individual depreciation induce consumer waiting behaviors, because consumers expect to benefit from a larger network as well as a lower period 2 selling price. Consequently, the vendor is able to charge a higher selling price p_2. (Indeed, it is straightforward to show that $\frac{\partial (p_2^*)^2}{\partial (1-\theta)\partial s} \geq 0$ when $\theta \in [0, \hat{\theta})$.) This, in turn, drives the positive interaction effect.

6.2. Conditional Pricing

In this section, we consider conditional pricing whereby the vendor commits to having a discount in period 2 for period 1 adopters, and compare this strategy with the selling model and the leasing model. We define conditional pricing as the following. At the beginning of period 1, the vendor commits that period 2 leasing fee (r_d) for period 1 adopters will be lower than period 2 leasing fee for new adopters (r_2), i.e., $r_d < r_2$. We have the following results.

Proposition 9. *For vintage depreciation information goods, conditional pricing is dominated by leasing.*

Proposition 10. *For individual depreciation information goods, when $\theta \in [0, \frac{1}{2})$, conditional pricing is weakly dominated by selling; otherwise, when $\theta \in [\frac{1}{2}, 1]$ conditional pricing is dominated by leasing.*

The intuition behind Proposition 9 is this. For vintage depreciation information goods and under leasing, the vendor does not incur any profit loss due to waiting, therefore conditional pricing cannot further improve vendor profit.

For individual depreciation information goods, when $\theta \in [\frac{1}{2}, 1]$, the existing adopters are willing to pay a period 2 leasing fee that is almost the same as the period 1 leasing fee. Thus conditional pricing (which offers a discounted price to existing adopters) is not needed. When $\theta \in [0, \frac{1}{2})$, while conditional pricing does improve profit, such improvement does not cause conditional pricing to dominate selling. Here is the intuition. Even at the discounted leasing fee r_d, not all existing adopters continue to lease in period 2 due to heavy individual depreciation. This coupled with the high period 2 leasing fee r_2, would cap the vendor's profit.

The above insight can be illustrated again using the two-consumer example in Section 3.2. Under leasing and individual depreciation, the optimal profit is 10 (see the right-bottom corner in Table 2). Now consider conditional pricing. Suppose $r_1 = 10$, $r_2 = 4$, and $r_d = 2.5$. In this case consumer V_1 has zero surplus if leasing in both periods. However, consumer V_1 has a surplus of 6 (i.e., $10 - 4 = 6$) if leasing only in period 2. Therefore the vendor must lower r_1 to 4 to attract consumer V_1's adoption in period 1. In REE, the optimal conditional pricing strategies are $r_1 = r_2 = 4$ and $r_d = 2.5$, and the vendor's profit is $4 + 4 + 2.5 = 10.5$. This example illustrates the case when conditional pricing dominates leasing, but is weakly dominated by selling.

As a remark, we stress here that such conditional pricing requires vendor commitment at the beginning of period 1. This is a fundamental departure from our baseline model assumption where we do not require such price commitment. It is well known in the literature that vendor price commitment is not desirable because of the vendor's inability to do so (e.g., Coase 1972, Bulow 1982, Katz and Shapiro 1986) – which is central to the time-inconsistency problem we discussed in the introduction section and in the literature review section.

6.3. The Hybrid Model

Finally, we briefly discuss the hybrid model, where consumers have the freedom to choose between purchasing and/or renting. We have the following Proposition 11.

Proposition 11. *For both vintage and individual depreciation information goods, in a two-period setting, the hybrid model cannot further increase the vendor's profit.*

We illustrate this somewhat surprising finding using our previous two-consumer example in Section 3.2. For brevity, we illustrate only with the case of individual depreciation. Similar examples can be constructed for the case of vintage depreciation. We first note that for the hybrid model to be optimal, it needs to be used by consumers. This in turn imposes the following structure in a two-period setting: one consumer will buy and the other will rent in period 1. Without loss of generality, let's assume in period 1 consumer V_1 chooses to buy in period 1, and consumer V_2 chooses to rent. In period 2, only consumer V_2 is left with a depreciated valuation of $4 \times 0.25 = 1$. Therefore the optimal period 2 leasing fee is 1 ($r_2^* = 1$). Then the optimal period 1 leasing fee is 1 ($r_1^* = 1$), otherwise consumer V_2 would delay her adoption. If the leasing fee is 1 in both periods, then the optimal selling price is 2 ($p_1^* = 2$). Otherwise consumer V_1 would choose to rent in both periods, rather than buying in period 1. Thus, the optimal total profit is 4 under the hybrid model, which is suboptimal as the vendor make more profits either using a pure selling strategy (10.5) or a pure leasing strategy (10).

The key intuition behind Proposition 11 is that offering the hybrid model does not further segment the market (thus resulting a higher profit). Rather, what it does is to offer additional incentives for consumers to wait, which in turn, does not improve profit further. For both types of consumer value depreciation, under REE, it is suboptimal to have both buying and renting consumers coexist in either period. In a two-period setting such as ours, selling and leasing models are profit equivalent in period 2. Then, both selling and leasing adopters must coexist in period 1 for the hybrid model to be optimal. This structure imposes an additional constraint to the selling price, $p_1 = r_1 + r_2$. Otherwise, if $p_1 > r_1 + r_2$, no consumers would choose to purchase in period 1(all in favor of renting); if $p_1 < r_1 + r_2$, no consumers would choose to rent in period 1 or in period 2 (all in favor of selling). Given this constraint, $p_1 = r_1 + r_2$, we see the hybrid model cannot further increase the vendor's profit, as it will never be chosen by any consumers. Note well here that Proposition 11 is obtained in a two-period setting assuming further no price commitment by the vendor *ex ante*[4]. We leave it for future research to study whether or not Proposition 11 can be extended to a T-period setting ($T > 2$) or to a continuous-time setting.

7. Conclusion

We examine the selling versus leasing debate for information goods in the context of valuation depreciation, using a two-period game-theoretic model. Our model considers two types of consumer-side valuation depreciation for information goods: vintage depreciation and individual depreciation. We are among the first to study individual depreciation information goods. While vintage depreciation has been studied in the durable goods literature for physical goods on the product side, we examine it in the context of information goods where we focus on the consumer-side valuation depreciation over time. Our extensions to consider network effects appear new in the literature, and we offer new insights. We find that, for vintage depreciation information goods, leasing dominates selling, with and without the presence of network effects. This seems to be consistent with some of the best business practices. For example, vendors of cloud storage services (e.g., Dropbox) and cloud computing services (e.g., Rackspace) often favor the leasing model. We would expect the network effects of the former is significant due to the nature of user collaboration, while the latter would have limited or no network effects. We then use the case of vintage depreciation as a benchmark for the case of individual depreciation, the main focus of our paper.

We find that leasing dominates selling when the magnitude of individual depreciation is small, which is consistent with real-world observations, such as Microsoft Office 365.[5] As yet another example, Wolfram Alpha, which is well known for its computing software solution Wolfram Mathematica, started to offer software on a subscription basis, starting with its version 8.[6]

[4] If we assume a comparable setup of a two-period model, a fixed leasing fee over time, and no production versioning (e.g., no product upgrading over time), our findings in Proposition 11 are consistent with prior literature (i.e., Zhang and Seidmann 2010).

[5] see http://www.microsoft.com/china/office365/buy.aspx

[6] http://www.wolfram.com/mathematica-home-edition/

Interestingly, we also find that selling dominates leasing when individual adopters heavily depreciate their valuation of the information goods, and this finding holds true in the presence of network effects as well. Furthermore, the interaction effect between individual depreciation and network effects can be either negative or positive. Our findings have immediate practical implications. For example, many mobile app games, such as "Angry Birds" or "Draw Something" receive a lot of attention because of their novelty, but then their novelty wears off, and users quickly get bored or distracted by similar products. In this case, selling could be more profitable than leasing. Amazon recently announced rental services for its Kindle digital book offerings (i.e., Kindle Unlimited). Our analytical results are not in favor of this pricing model migration, which is evident by Amazon's limited offerings in Kindle Unlimited, especially in certain categories with strong individual depreciation[7].

Our model results also suggest an interesting strategic interaction between individual depreciation (i.e., θ) and network effects (i.e., s). For vendors of information goods, the optimal pricing scheme can be obtained by locating themselves in Figure 7 with their parameter pair of θ and s. For example, sales of *individual* mp music titles rely on the selling model only, due to their significant individual depreciation (e.g., iTunes). However, the leading music streaming service website Spotify has been very successful in using the periodic subscription pricing model. It offers more than 20 million songs, produced by multiple publishers, with weekly updates. Linking this observation to our model, we posit that Spotify's offering of a package of songs, rather than a single song, is relatively less sensitive to individual depreciation, making the leasing model more favorable. Spotify's offering of music collection and Amazon's Kindle Unlimited, represent examples of how vendors of information goods can change individual depreciation through bundling. In addition, vendors of similar products may choose different pricing schemes depending on the strength of network effects. Take computer games as an example. The leasing model is popular among the network-based versions of games (e.g., Blizzard's World of Warcraft) when network effects are strong; on the other

[7] As of August 2016, 29.2% of ebook titles in Amazon Kindle Store (1,372,503 out of 4,703, 075 titles) are eligible for Kindle Unlimited. Some categories with strong individual depreciation, such as non-fiction novels, are not included in the Kindle Unlimited program.

hand, selling becomes more favorable when network effects are weak (e.g., the single-player game Warcraft III).

For future research, it would be interesting to extend our model to the multiple-period or continuous-time settings. Another fruitful avenue of future research would be testing our model predictions empirically.

References

Bagnoli, M., S.W. Salant, J.E. Swierzbinski. 1989. Durable-goods monopoly with discrete demand. *Journal of Political Economy* **97**(6) 1459–1478.

Bensaid, Bernard. 1996. Dynamic monopoly pricing with network externalities. *International Journal of Industrial Organization* **14**(6) 837–855.

Bhargava, H.K., R.R. Chen. 2012. The benefit of information asymmetry: When to sell to informed customers? *Decision Support Systems* **53**(3) 345–356.

Bhargava, H.K., S. Sundaresan. 2004. Computing as utility: Managing availability, commitment, and pricing through contingent bid auctions. *Journal of Management Information Systems* **21**(2) 201–227.

Bhaskaran, Sreekumar R., Stephen M. Gilbert. 2005. Selling and leasing strategies for durable goods with complementary products. *Management Science* **51**(8) 1278–1290.

Bhaskaran, Sreekumar R., Stephen M. Gilbert. 2009. Implications of channel structure for leasing or selling durable goods. *Marketing science* **28**(5) 918–934.

Bond, Eric W., Larry Samuelson. 1984. Durable good monopolies with rational expectations and replacement sales. *The RAND Journal of Economics* **15**(3) 336–345.

Bulow, J.I. 1982. Durable goods monopolists. *Journal of Political Economy* **90**(2) 314–332.

Chien, H.K., C.Y.C. Chu. 2008. Sale or lease? durable-goods monopoly with network effects. *Marketing Science* **27**(6) 1012–1019.

Choudhary, V. 2007. Software as a service: Implications for investment in software development. *Proceedings of the 40th Hawaii International Conference on System Sciences*. IEEE Computer Society.

Coase, R.H. 1972. Durability and monopoly. *Journal of Law and Economics* **15**(1) 143–149.

Conlisk, J.E., E. Gerstner, J. Sobel. 1984. Cyclic pricing by a durable goods monopolist. *Quarterly Journal of Economics* **99**(3) 489–505.

Conner, K.R. 1995. Obtaining strategic advantage from being imitated: When can encouraging "clones" pay? *Management Science* **41**(2) 209–225.

Desai, P., D. Purohit. 1998. Leasing and selling: Optimal marketing strategies for a durable goods firm. *Management Science* **44**(11) S19–S34.

Farrell, J., G. Saloner. 1986. Installed base and compatibility: Innovation, product preannouncements, and predation. *American Economic Review* **76**(5) 940–955.

Fudenberg, D., J. Tirole. 1991. *Game Theory*. MIT Press, Cambrige, Boston, U.S.

Gul, Faruk, Hugo F. Sonnenschein, Robert Wilson. 1986. Foundations of dynamic monopoly and the coase conjecture. *Journal of Economic Theory* **39**(1) 155–190.

Hu, J.Y. 2005. Essays on internet markets and information goods. Ph.D. thesis, MIT.

Huang, K.W., A. Sundararajan. 2005. Pricing models for on-demand computing. *Working paper, Stern School of Business, New York University* .

Ishihara, Masakazu, Andrew Ching. 2012. Dynamic demand for new and used durable goods without physical depreciation: The case of japanese video games. *Working Paper, Rotman School of Management, University of Toronto* .

Jain, S., P.K. Kannan. 2002. Pricing of information products on online servers: issues, models, and analysis. *Management Science* **48**(9) 1123–1142.

Jing, Bing. 2007. Network externalities and market segmentation in a monopoly. *Economics Letters* **95**(1) 7–13.

Johnson, J.P., D. P. Myatt. 2003. Multiproduct quality competition: Fighting brands and product line pruning. *American Economic Review* **93**(3) 748–774.

Johnson, J.P., D. P. Myatt. 2006. On the simple economics of advertising, marketing, and product design. *American Economic Review* **96**(3) 756–784.

Katz, M.L., C. Shapiro. 1985. Network externalities, competition, and compatibility. *American Economic Review* **75**(3) 424–440.

Katz, M.L., C. Shapiro. 1986. Technology adoption in the presence of network externalities. *Journal of Political Economy* **94**(4) 822–841.

Katz, M.L., C. Shapiro. 1992. Product introduction with network externalities. *Journal of Industrial Economics* **40**(1) 55–83.

Shapiro, C., H.R. Varian. 1999. *Information Rules: A Strategic Guide to the Network Economy*. Harvard Business Press, Boston, MA.

Shiller, Benjamin R. 2013. Digital distribution and the prohibition of resale markets for information goods. *Quantitative Marketing and Economics* **11**(4) 403–435.

Stokey, Nancy L. 1979. Intertemporal price discrimination. *Quarterly Journal of Economics* **93**(3) 355–371.

Suslow, Valerie Y. 1986. Commitment and monopoly pricing in durable goods models. *International Journal of Industrial Organization* **4**(4) 451–460.

Swan, Peter L. 1970. Durability of consumption goods. *American Economic Review* **60**(5) 884–894.

Zhang, J., A. Seidmann. 2010. Perpetual versus subscription licensing under quality uncertainty and network externality effects. *Journal of Management Information Systems* **27**(1) 39–68.

Appendix

Proofs

Proof of Proposition 2. Denote $SW^{S,A}$ as the social welfare for vintage depreciation information goods (A) under the selling model (S). We have

$$SW^{S,A} = \int_{v_1}^{1} (1+\theta)v\mathrm{d}v + \int_{v_2}^{v_1} \theta v\mathrm{d}v.$$

Since $v_1 = 1 - N_1^*$ and $v_2 = 1 - N_1^* - N_2^*$, and N_1^* and N_2^* are given by Equation (3), solving, we have $v_1 = \frac{2+\theta}{4+\theta}$ and $v_2 = \frac{2+\theta}{2(4+\theta)}$. Inserting v_1 and v_2 into $SW^{S,A}$ gives

$$SW^{S,A} = \int_{\frac{2+\theta}{4+\theta}}^{1} (1+\theta)v\mathrm{d}v + \int_{\frac{2+\theta}{2(4+\theta)}}^{\frac{2+\theta}{4+\theta}} \theta v\mathrm{d}v = \frac{4+3\theta}{8} - \frac{1}{2(4+\theta)}.$$

Similarly, the social welfare for vintage depreciation information goods (A) under the leasing model (L), denoted as $SW^{L,A}$, is

$$SW^{L,A} = \int_{v_1}^{1} v\mathrm{d}v + \int_{v_2}^{1} \theta v\mathrm{d}v = \int_{\frac{1}{2}}^{1} v\mathrm{d}v + \int_{\frac{1}{2}}^{1} \theta v\mathrm{d}v = \frac{3(1+\theta)}{8}.$$

Proposition 2 holds because $SW^{S,A} - SW^{L,A} = \frac{\theta}{8(4+\theta)} \geq 0$. $\square$

Proof of Lemma 1. This is the case for individual depreciation information goods (B) under the selling model (S). We prove via backward induction. At the beginning of period 2, potential adopter types are located on $[0, v_1]$. The marginal consumer type in period 2, v_2, satisfies $v_2 = p_2$. The selling vendor's period 2 profit is $p_2(v_1 - v_2)$. Optimizing the vendor's profit gives, $p_2^* = \frac{v_1}{2}$ and $\pi_2^* = \frac{v_1^2}{4}$.

Next consider the vendor's problem in period 1. The marginal consumer type in period 1, v_1, satisfies the following following equation,

$$U_{v_1}^{S,B}(DD) = U_{v_1}^{S,B}(OD) \Rightarrow v_1 + \theta v_1 - p_1 = v_1 - p_2^*. \tag{A.1}$$

Inserting $p_2^* = \frac{v_1}{2}$ into Equation (A.1) results $p_1 = \frac{(1+2\theta)v_1}{2}$.

At the beginning of period 1, the vendor's profit is $\pi^{S,B} = p_1(1-v_1) + \pi_2^*(v_1) = \frac{v_1[2(1+2\theta)-(1+4\theta)v_1]}{4}$, which is concave in v_1. Solving, we have $v_1^* = \frac{1+2\theta}{1+4\theta}$. Lemma 1 follows immediately. □

Proof of Lemma 2. Consider period 2. At the beginning of period 2, period 1 adopters are distributed in the interval $(v_1, 1]$. Under individual depreciation, all consumers are potential adopters in period 2. There are three scenarios to consider in period 2.

Scenario 1: $v_1 = 1$;

Scenario 2: $v_1 < 1$, $v_1 < \theta$;

Scenario 3: $v_1 < 1$, $v_1 \geq \theta$.

Scenario 1 implies that there are no adopters in period 1. In this case, period 2 marginal consumer type v_2 satisfies $v_2 = r_2$, and period 2 profit is $r_2(1-v_2)$. Solving the vendor's period 2 profit optimization problem gives $v_2^* = \frac{1}{2}$ and $\pi_2^* = \frac{1}{4}$.

Under scenario 2, there are three pricing regions in period 2: (1) $v_2 \leq \theta v_1$, (2) $v_2 \in (\theta v_1, v_1)$, and (3) $v_2 \in [v_1, \theta]$. We first rule out the second price region which is infeasible according to the following Lemma A1.

Lemma A1. *For individual depreciation information goods and under the leasing model, given any rental price pair* (r_1, r_2)*, if there are simultaneous period 1 only and period 2 only adopters, then* (r_1, r_2) *does not constitute a REE.*

We prove Lemma A1 by contradiction. Assume that consumers correctly anticipate the rental price pair (r_1, r_2), and period 1 only and period 2 only adopters coexist in the market. The surplus for a type-v period 1 only adopter is $v - r_1$. Similarly, the surplus for a type-v period 2 only adopter is $v - r_2$. Rational consumers will rent only in the period with the smaller rental price, unless $r_1 = r_2$ (in which case there is no decision for the vendor to make in period 2). Contradiction.

Lemma A1 thus implies that only the first and third price regions are feasible for scenario 2. In price region (1), the optimal period 2 rental price is obtained by maximizing period 2 profit $r_2(1-v_2)$, subject to $r_2 < \theta v_1$ and $r_2 = v_2$. The optimal solutions and profit are

$$r_2^* = \begin{cases} \frac{1}{2}, & v_1 \geq \frac{1}{2\theta}; \\ \theta v_1, & v_1 < \frac{1}{2\theta}. \end{cases}, \quad \pi_2^* = \begin{cases} \frac{1}{4}, & v_1 \geq \frac{1}{2\theta}; \\ \theta v_1(1-\theta v_1), & v_1 < \frac{1}{2\theta}. \end{cases} \tag{A.2}$$

Similarly, the optimal rental fee r_2^* and the corresponding profit π_2^* in price region (3) are

$$r_2^* = \begin{cases} v_1, & v_1 \geq \frac{\theta}{2}; \\ \frac{\theta}{2}, & v_1 < \frac{\theta}{2}. \end{cases}, \quad \pi_2^* = \begin{cases} v_1(1-\frac{v_1}{\theta}), & v_1 \geq \frac{\theta}{2}; \\ \frac{\theta}{4}, & v_1 < \frac{\theta}{2}. \end{cases}$$

In scenario 3, there is only one feasible price region, $v_2 \leq \theta v_1$. Therefore the solution is identical to Equation (A.2), and we have a total of five candidate strategies for period 1. We must combine them for the optimal period 2 strategies under all possible value of v_1.

Period 2, candidate strategy 1: $v_1 = 1$, $r_2 = \frac{1}{2}$ and $\pi_2 = \frac{1}{4}$;

Period 2, candidate strategy 2: $v_1 \in [\frac{1}{2\theta}, 1)$, $r_2 = \frac{1}{2}$ and $\pi_2 = \frac{1}{4}$;

Period 2, candidate strategy 3: $v_1 < \frac{1}{2\theta}$, $r_2 = \theta v_1$ and $\pi_2 = \theta v_1(1-\theta v_1)$;

Period 2, candidate strategy 4: $v_1 < \frac{\theta}{2}$, $r_2 = \frac{\theta}{2}$ and $\pi_2 = \frac{\theta}{4}$;

Period 2, candidate strategy 5: $v_1 \in [\frac{\theta}{2}, \theta]$, $r_2 = v_1$ and $\pi_2 = v_1(1-\frac{v_1}{\theta})$.

Combining all five candidate strategies leads to the following period 2 optimal strategies for different regions of v_1 and θ:

Region 1: $v_1 = 1$, $r_2 = \frac{1}{2}$, and $\pi_2 = \frac{1}{4}$;

Region 2: $v_1 \in [\frac{1}{2\theta}, 1)$ and $\theta \in [\frac{1}{2}, 1]$, $r_2 = \frac{1}{2}$, and $\pi_2 = \frac{1}{4}$;

Region 3: $v_1 \in [\frac{\theta}{1+\theta+\theta^2}, 1)$ and $\theta \in [0, \frac{1}{2})$, $r_2 = \theta v_1$, and $\pi_2 = \theta v_1(1-\theta v_1)$;

Region 4: $v_1 \in [\frac{\theta}{1+\theta+\theta^2}, \frac{1}{2\theta})$ and $\theta \in [\frac{1}{2}, \frac{\sqrt{5}-1}{2})$, $r_2 = \theta v_1$, and $\pi_2 = \theta v_1(1-\theta v_1)$;

Region 5: $v_1 \in [\frac{1-\sqrt{1-\theta}}{2\theta}, \frac{1}{2\theta})$ and $\theta \in [\frac{\sqrt{5}-1}{2}, 1]$, $r_2 = \theta v_1$, and $\pi_2 = \theta v_1(1-\theta v_1)$;

Region 6: $v_1 \in [0, \frac{\theta}{2})$ and $\theta \in [0, \frac{\sqrt{5}-1}{2})$, $r_2 = \frac{\theta}{2}$, and $\pi_2 = \frac{\theta}{4}$;

Region 7: $v_1 \in [\frac{1-\sqrt{1-\theta}}{2\theta}, \frac{1}{2\theta})$ and $\theta \in [\frac{\sqrt{5}-1}{2}, 1]$, $r_2 = \frac{\theta}{2}$, and $\pi_2 = \frac{\theta}{4}$;

Region 8: $v_1 \in [\frac{\theta}{2}, \frac{\theta}{1+\theta+\theta^2})$ and $\theta \in [0, \frac{\sqrt{5}-1}{2})$, $r_2 = v_1$, and $\pi_2 = v_1(1-\frac{v_1}{\theta})$.

In period 1, we move through Regions 1 to 8 above to solve for v_1^*. There are a total of six candidate strategies in period 1:

Period 1, candidate strategy 1: $v_1 = 1$, and $\pi = \frac{1}{4}$;

Period 1, candidate strategy 2: $\theta \in [\frac{1}{2}, 1]$, $v_1 = \frac{1}{2\theta}$, and $\pi = \frac{3}{4} - \frac{1}{4\theta}$;

Period 1, candidate strategy 3: $v_1 = \frac{1}{1+\theta}$, and $\pi = \frac{\theta}{1+\theta}$;

Period 1, candidate strategy 4: $\theta \in [0, \frac{\sqrt{5}-1}{2})$, $v_1 = \frac{\theta}{2}$, and $\pi = \frac{\theta(3-\theta)}{4}$;

Period 1, candidate strategy 5: $\theta \in [\frac{\sqrt{5}-1}{2}, 1]$, $v_1 = \frac{1-\sqrt{1-\theta}}{2\theta}$, and $\pi = \frac{\theta^3+2[(1-\theta)\sqrt{1-\theta}-1]+3\theta}{4\theta^2}$;

Period 1, candidate strategy 6: $\theta \in [0, \frac{\sqrt{5}-1}{2})$, $v_1 = \frac{\theta}{1+\theta+\theta^2}$, and $\pi = \frac{\theta(1+\theta+2\theta^2)}{(1+\theta+\theta^2)^2}$.

The optimal solution in Lemma 2 can be obtained by comparing all candidate strategies in period 1. □

Proof of Proposition 4. Denote $SW^{S,B}$ as the social welfare for individual depreciation information goods (B) under the selling model (S). From Lemma 1, we have

$$SW^{S,B} = \int_{v_1}^{1} (1+\theta)v\mathrm{d}v + \int_{v_2}^{v_1} v\mathrm{d}v = \frac{3+4\theta[9+\theta(19+12\theta)]}{8(1+4\theta)^2}.$$

Denote $SW^{L,B}$ as the social welfare for individual depreciation information goods (B) under the leasing model (L). From Lemma 2, we have the following two cases:

Case 1: $\theta < \frac{1}{3}$. Then there are only period 2 adopters and $v_2 = \frac{1}{2}$, which gives that $SW^{L,B} = \int_{\frac{1}{2}}^{1} v\mathrm{d}v = \frac{3}{8} < SW^{S,B}$;

Case 2: $\theta \geq \frac{1}{3}$. $v_1 = \frac{1}{1+\theta}$ and $v_2 = \frac{\theta}{1+\theta}$. All period 1 adopters continue adopting in period 2. Then

$$SW^{L,B} = \int_{v_1}^{1} (1+\theta)v\mathrm{d}v + \int_{v_2}^{v_1} v\mathrm{d}v = \frac{1}{2} + \frac{\theta^2}{2(1+\theta)} < SW^{S,B}.$$

Thus, in either case, $SW^{L,B} < SW^{S,B}$. □

Proof of Proposition 5. We first derive a selling vendor's profit.

First, consider period 2. The marginal consumer type v_2 satisfies $p_2 = \theta(v_2 + s\frac{1-v_2}{1+K})$. Period 2 profit is $\frac{p_2(v_1-v_2)}{1+K} = \frac{\theta(v_1-v_2)(s+(1+K-s)v_2)}{(1+K)^2}$, which is always concave in v_2. The interior optimal solution is $v_2^* = \frac{1}{2}\left[v_1 - \frac{s}{1+K-s}\right]$, and p_2^* is nonnegative when $v_1 \geq -\frac{s}{1+K-s}$. Therefore, for $v_1 \geq -\frac{s}{1+K-s}$, $p_2^* = \frac{\theta(s+(1+K-s)v_1)}{2(1+K)}$, and $(\tilde{\pi}_2^{S,A})^* = \frac{\theta(s+(1+K-s)v_1)^2}{4(1+K)^2(1+K-s)}$. Otherwise $v_2^* = v_1$.

Next, consider period 1. The marginal consumer v_1 is indifferent between buying in period 1 and buying in period 2, which implies

$$v_1 + sN_1 - p_1 + \theta(v_1 + s(N_1 + N_2))^+ = \theta(v_1 + s(N_1 + N_2))^+ - p_2.$$

This leads to $p_1 = \frac{(2+\theta)[s+(1+K-s)v_1]}{2(1+K)}$. Inserting p_1 back into the profit function $p_1(1-v_1) + (\tilde{\pi}_2^{S,A})^*$ gives

$$\tilde{\pi}^{S,A} = \frac{2s(2+\theta)(1+2K) - s^2(4+\theta)}{4(1+K)^2(1+K-s)} - \frac{2(1+K-s)v_1(2s(4+\theta) - 2(1+K)(2+\theta))}{4(1+K)^2(1+K-s)} - \frac{(1+K-s)(4+\theta)v_1^2}{4(1+K)^2},$$

which is concave in v_1. The interior solution is

$$v_1^* = 1 - \frac{2(1+K)}{(1+K-s)(4+\theta)}.$$

This interior solution is obtainable because $v_1^* > -\frac{s}{1+K-s}$ always hold. The optimal profit is $\frac{(2+\theta)^2}{4(1+K-s)(4+\theta)}$.

We now derive a leasing vendor's profit.

The marginal consumer type v_2 satisfies $r_2 = \theta\left(v_2 + \frac{s(1-v_2)}{1+K}\right)$. Inserting this back into the profit function $\frac{r_2(1-v_2)}{1+K}$ gives

$$\pi_2 = \frac{\theta(1-v_2)[s + (1+K-s)v_2]}{(1+K)^2},$$

which is concave in v_2. The interior solution is $v_2^* = \frac{1+K-2s}{2(1+K-s)}$, and the optimal period 2 profit is $(\tilde{\pi}_2^{L,A})^* = \frac{\theta}{4(1+K-s)}$.

In period 1, the marginal consumer type v_1 satisfies $v_1 + sN_1 - r_1 = 0$, which implies that consumers are purely myopic. Using an argument similar to that in period 2, we can obtain that $v_1^* = v_2^* = \frac{1+K-2s}{2(1+K-s)}$. The profit is $\frac{1+\theta}{4(1+K-s)}$.

Leasing dominates selling in profit, because $\frac{1+\theta}{4(1+K-s)} > \frac{(2+\theta)^2}{4(1+K-s)(4+\theta)}$ holds for all $\theta \in [0,1]$. $\square$

Proof of Proposition 6. This proof is similar to the proofs of Lemma 1 and Lemma 2. For brevity, we only provide a sketch of the proof here. A detailed proof is available upon request.

Individual depreciation: Selling

Under the selling model, in period 2, when $v_1 \geq -\frac{s}{1+K-s}$, there exists a unique interior solution such that $v_2 = \frac{1}{2}\left(v_1 - \frac{s}{1+K-s}\right)$ and the optimal period 2 profit is $\frac{[s+(1+K-s)v_1]^2}{4(1+K)^2(1+K-s)}$.

In period 1, the marginal consumer type v_1 satisfies

$$v_1 + sN_1 - p_1 + \theta(v_1 + s(N_1 + N_2)) = v_1 + s(N_1 + N_2) - p_2.$$

Inserting N_2^* and p_2^* into this equation and rearranging, we obtain:

$$p_1 = \frac{(s(1-v_1) + v_1(1+K))(1+K-s\theta - 2(s-(1+K)\theta))}{2(1+K)(1+K-s)}.$$

We now have two cases to consider:

Case 1: $s \geq \frac{(1+K)(1+4\theta)}{3+2\theta}$, the profit function is convex in v_1, $v_1^* = 1$, and $(\tilde{\pi}^{S,B})^* = \frac{1}{4(1+K-s)}$;

Case 2: $s < \frac{(1+K)(1+4\theta)}{3+2\theta}$, the profit function is concave in v_1, interior solution is $v_1^* = 1 - \frac{1+K}{2}\left(\frac{1}{1+K-s} - \frac{1}{1+K-3s+4\theta+4K\theta-2s\theta}\right)$, which is obtainable when $s \leq \frac{2\theta(1+K)}{1+\theta}$.

Combining case 1 and 2, we have: When $s \leq \frac{2\theta(1+K)}{1+\theta}$, $v_1^* = 1 - \frac{1+K}{2}\left(\frac{1}{1+K-s} - \frac{1}{1+K-3s+4\theta+4K\theta-2s\theta}\right)$, and $(\tilde{\pi}^{S,B})^* = \frac{((1+K)(1+2\theta)-s(2+\theta))^2}{4(1+K-s)^2((1+K)(1+4\theta)-s(3+2\theta))}$; otherwise when $s > \frac{2\theta(1+K)}{1+\theta}$, $v_1^* = 1$ and $(\tilde{\pi}^{S,B})^* = \frac{1}{4(1+K-s)}$.

Individual depreciation: Leasing

Under the leasing model, similar to the proof of Lemma 2, we consider multiple scenarios for period 2 and obtain the following candidate strategies:

Candidate strategy 1: $v_1 = 1$, $v_2 = \frac{1+K-2s}{2(1+K-s)}$ and $(\tilde{\pi}_2^{L,B})^* = \frac{1}{4(1+K-s)}$;

Candidate strategy 2: $v_1 \geq \frac{1+K-s(1+\theta)}{2(1+K-s)\theta}$, $v_2 = \frac{1+K-2s}{2(1+K-s)}$, and $(\tilde{\pi}_2^{L,B})^* = \frac{1}{4(1+K-s)}$;

Candidate strategy 3: $v_1 < \frac{1+K-s(1+\theta)}{2(1+K-s)\theta}$, $v_2 = \frac{(1+K)\theta v_1 - s(1-\theta)}{1+K-s(1-\theta)}$, and $(\tilde{\pi}_2^{L,B})^* = \frac{\theta[s+(1+K-s)v_1](1-\theta v_1)}{(1+K-s(1-\theta))^2}$;

Candidate strategy 4: $v_1 \geq \frac{\theta}{2} - \frac{s}{2(1+K-s)}$, $v_2 = \frac{s(1-\theta)+v_1(1+K)}{s(1-\theta)+\theta(1+K)}$, and $(\tilde{\pi}_2^{L,B})^* = \frac{\theta[s+(1+K-s)v_1](1-\theta v_1)}{(1+K-s(1-\theta))^2}$;

Candidate strategy 5: $v_1 < \frac{\theta}{2} - \frac{s}{2(1+K-s)}$, $v_2 = \frac{1+K-2s}{2(1+K-s)}$, and $(\tilde{\pi}_2^{L,B})^* = \frac{\theta}{4(1+K-s)}$.

Combining all these candidate strategies produces the optimal period 2 solution, which contains seven regions. In period 1, we search through all seven regions for an optimal solution that satisfies the REE criteria. The optimal period 1 solution contains at most four candidate solutions, each of which covers a certain parameter space (subregions) of (θ, s).

In region $s \geq \frac{(1+K)(1-2\theta)}{1-\theta}$, leasing dominates selling; In region $s \geq \frac{2\theta(1+K)}{1+\theta}$, these two pricing models converge, with $v_1 = 1$ and $(\tilde{\pi}^{L,B})^* = \frac{1}{4(1+K-s)}$; In all other regions, selling dominates leasing. $\square$.

Proof of Proposition 7. $\tilde{\pi}^* = \frac{1+\theta}{4(1+K-s)}$. $\frac{\partial \tilde{\pi}^*}{\partial s} = \frac{1+\theta}{4(1+K-s)^2} > 0$, $\frac{\partial^2 \tilde{\pi}^*}{\partial s^2} = \frac{1+\theta}{4(1+K-s)^3} > 0$.

$\frac{\partial^2 \tilde{\pi}^*}{\partial\theta\partial s} = \frac{1}{4(1+K-s)^2} > 0$ which means $\frac{\partial^2 \tilde{\pi}^*}{\partial(1-\theta)\partial s} < 0$. □

Proof of Proposition 8. There are three regions to consider:

Region 1: $\theta \in [0, \frac{s}{2+2K-s})$. $\tilde{\pi}^* = \frac{1}{4(1+K-s)}$. $\frac{\partial\tilde{\pi}^*}{\partial s} = \frac{1}{4(1+K-s)^2} > 0$, $\frac{\partial^2\tilde{\pi}^*}{\partial s^2} = \frac{1}{4(1+K-s)^3} > 0$ and $\frac{\partial^2\tilde{\pi}^*}{\partial\theta\partial s} = 0$.

Region 2: $\theta \in [\frac{s}{2+2K-s}, \frac{1+K-s}{2+2K-s})$. $\tilde{\pi}^* = \frac{((1+2\theta)(1+K)-(\theta+2)s)^2}{4(K-s+1)^2((1+4\theta)(1+K)-(2\theta+3)s)}$.

$$\begin{aligned}\frac{\partial\tilde{\pi}^*}{\partial s} &= (1+K+2\theta+2K\theta-s(2+\theta)) \\ &\times \frac{(1+K)(1+K-3s)+6s^2+\left(2(1+K)^2-17(1+K)s+7s^2\right)\theta+2\theta^2(1+K-2s)(1+K-3s)}{4(1+K-s)^3((1+K)(1+4\theta)-s(3+2\theta))^2},\end{aligned}$$

which is positive.

$$\begin{aligned}\frac{\partial^2\tilde{\pi}^*}{\partial s^2} &= \frac{1}{8}\left(-\frac{1+K+5s}{(1+K-s)^4}+\frac{2(4+4K-s)\theta}{(1+K-s)^4}+\frac{25(1+K)^2}{(-2-2K+s)^2(1+K+4\theta+4K\theta-s(3+2\theta))^3}\right) \\ &+ \frac{10(1+K)}{8(2+2K-s)^2(1+K+4\theta+4K\theta-s(3+2\theta))^2}+\frac{1}{8(2+2K-s)^2(1+K+4\theta+4K\theta-s(3+2\theta))},\end{aligned}$$

which is positive.

$$\frac{\partial^2\tilde{\pi}^*}{\partial s\partial\theta} = \frac{1}{8}\left(\frac{3+3K-s}{(1+K-s)^3}-\frac{1}{(1+K-3s+4\theta+4K\theta-2s\theta)^2}-\frac{10(1+K)}{(1+K+4\theta+4K\theta-s(3+2\theta))^3}\right),$$

which is not always positive. However, we can show that $\frac{\partial^2\tilde{\pi}^*}{\partial s\partial\theta}$ is increasing in θ, which implies that there exists a unique $\hat{\theta}$ such that the cross-derivative is negative for $\theta \in [\frac{s}{2+2K-s}, \hat{\theta})$ and nonnegative for $\theta \in [\hat{\theta}, \frac{1+K-s}{2+2K-s})$. $\hat{\theta}$ is the unique solution to the equation $\frac{\partial^2\tilde{\pi}^*}{\partial s\partial\theta} = 0$.

Region 3: $\theta \in [\frac{1+K-s}{2+2K-s}, 1]$. $\tilde{\pi}^* = \frac{(2(1+K)\theta-s(1-\theta))^2}{4(1+K-s)\left(s^2(1-\theta)^2+(1+K)^2\theta(1+\theta)-(1+K)s(1-\theta^2)\right)}$.

$$\frac{\partial\tilde{\pi}^*}{\partial s} = \frac{(2\theta(K+1)+(\theta-1)s)\left(2\theta^2(\theta+1)(K+1)^3+6(\theta-1)^2\theta(K+1)s^2+(\theta-1)(7\theta-1)(K+1)^2s+(\theta-1)^3s^3\right)}{4(K-s+1)^2\left(\theta(\theta+1)(K+1)^2+(\theta^2-1)(K+1)s+(\theta-1)^2s^2\right)^2},$$

which is positive for $\theta \in [\frac{1+K-s}{2+2K-s}, 1]$.

$$\begin{aligned}\frac{\partial^2\tilde{\pi}^*}{\partial s^2} &= \frac{1}{2(K-s+1)^3\left(\theta(\theta+1)(K+1)^2+(\theta^2-1)(K+1)s+(\theta-1)^2s^2\right)^3} \\ &\times \left(\begin{array}{l}12\theta(\theta-1)^5(K+1)s^5+12\theta^3(\theta+1)^2(\theta-1)(K+1)^5s+(\theta-1)^6s^6 \\ +\theta^2(\theta+1)(\theta(\theta(\theta+11)-5)+1)(K+1)^6+3(\theta-1)^4(\theta+1)(8\theta-1)(K+1)^2s^4 \\ +(\theta-1)^3(\theta(\theta(7\theta+60)+3)-2)(K+1)^3s^3+6(\theta-1)^2\theta(\theta(\theta(4\theta+3)+6)-1)(K+1)^4s^2\end{array}\right),\end{aligned}$$

which is positive.

$$\frac{\partial^2 \tilde{\pi}^*}{\partial s \partial \theta} = \frac{1+K}{4(K-s+1)^2 \left(\theta(\theta+1)(K+1)^2+(\theta^2-1)(K+1)s+(\theta-1)^2 s^2\right)^3}$$

$$\times \begin{pmatrix} 4\theta^3(\theta+1)(K+1)^5-(\theta-1)^3(\theta+11)(K+1)s^4+7(\theta-1)^3(5\theta+1)(K+1)^2 s^3-4(\theta-1)^4 s^5 \\ +(\theta-1)\theta(\theta(7\theta+32)-31)(K+1)^3 s^2-2\theta(\theta(\theta(7\theta-17)+9)+1)(K+1)^4 s) \end{pmatrix},$$

which is positive. □

Proof of Proposition 9. For vintage depreciation information goods and under leasing, as illustrated by Figure 3, the vendor does not suffer from consumers' waiting behaviors because consumers do not need to take the future price r_2 into consideration. Consequently, conditional pricing does not impact period 1 adoption, committing $r_d < r_2$ can only decreases the vendor's profit. □

Proof of Proposition 10. This proof is similar to the proof of Lemma 2. We only provide a sketch here. Period 2 optimal solutions include 5 different regions:

Region 1: $\theta < \frac{1}{2}$, $v_1 \in [0, \frac{2\theta-\sqrt{\theta}}{4\theta-1})$, $\pi_2^* = \frac{\theta}{4}$;

Region 2: $\theta < \frac{1}{2}$, $v_1 \in [\frac{2\theta-\sqrt{\theta}}{4\theta-1}, 1]$, $\pi_2^* = \theta v_1(1-v_1) + \frac{v_1^2}{4}$;

Region 3: $\theta \geq \frac{1}{2}$, $v_1 \in [0, \frac{1-\sqrt{1-\theta}}{2\theta})$, $\pi_2^* = \frac{\theta}{4}$;

Region 4: $\theta \geq \frac{1}{2}$, $v_1 \in [\frac{1-\sqrt{1-\theta}}{2\theta}, \frac{1}{2\theta})$, $\pi_2^* = \theta v_1(1-\theta v_1)$;

Region 5: $\theta \geq \frac{1}{2}$, $v_1 \in [\frac{1}{2\theta}, 1]$, $\pi_2^* = \frac{1}{4}$.

In each region, we solve for v_1^* that satisfies REE.

When $\theta \in [0, \frac{1}{2})$, the optimal pricing strategies are $r_1^* = r_2^* = \frac{1+2\theta}{2(1+4\theta)}$, and $r_d^* = \frac{\theta}{2}$. The optimal profit is $\frac{(1+2\theta)^2}{4(1+4\theta)}$, which is the same as the selling vendor's profit. Thus, conditional pricing is weakly dominated by selling. When $\theta \in [\frac{1}{2}, 1]$, the optimal pricing strategies are $r_1^* = r_2^* = \frac{\theta}{1+\theta}$, and $r_d^* \to r_2^*$. Thus, conditional pricing (committing $r_d < r_2$) is suboptimal and is dominated by leasing. □

Proof of Proposition 11. We start by proving the following Lemma A2.

Lemma A2. *In a two-period setting, under either vintage or individual depreciation, period 1 only consumers and period 2 only consumers can never co-exist.*

We prove Lemma A2 by contradiction. Assume period 1 only consumers and period 2 only consumers can co-exist.

Case 1: individual depreciation. Without loss of generality, assume that there exists a type-v consumer who chooses the leasing model only in period 1, i.e., $U_v^{L,B}(DO) > U_v^{L,B}(OD)$. This leads to $v - r_1 > v - r_2 \Rightarrow r_2 > r_1$. By assumption, there exists another type-v' consumer, who also chooses the leasing model but adopt only in period 2, i.e., $U_{v'}^{L,B}(OD) > U_{v'}^{L,B}(DO)$. This leads to $v' - r_1 < v' - r_2 \Rightarrow r_2 < r_1$. Contradiction.

Case 2: vintage depreciation. W.o.l.g., assume that there exists a type-v consumer who chooses leasing and adopts only in period 1, this gives

$$\begin{aligned} U_v^{L,A}(DO) > U_v^{L,A}(OD) \ &\Rightarrow v - r_1 > \theta v - r_2 \Rightarrow r_1 - r_2 < (1-\theta)v; \\ U_v^{L,A}(DO) > U_v^{L,A}(DD) \ &\Rightarrow v - r_1 > v - r_1 + \theta v - r_2 \Rightarrow r_2 > \theta v. \end{aligned}$$

By assumption there exists another type-v' consumer who chooses leasing and adopt only in period 2, this gives

$$\begin{aligned} U_{v'}^{L,A}(OD) > U_{v'}^{L,A}(DO) \ &\Rightarrow \theta v' - r_2 > v' - r_1 \Rightarrow r_1 - r_2 > (1-\theta)v; \\ U_{v'}^{L,A}(OD) > U_{v'}^{L,A}(OO) \ &\Rightarrow \theta v' - r_2 > 0 \Rightarrow r_2 < \theta v'. \end{aligned}$$

Combining the above four conditions, we have

$$(1-\theta)v' < r_1 - r_2 < (1-\theta)v \Rightarrow v > v';$$

$$\theta v < r_2 < \theta v' \Rightarrow v' > v.$$

Contradiction.

We now prove Proposition 11 by contradiction. Assume that the hybrid model is optimal. This requires that both the selling model and the leasing model are adopted simultaneously, by different market segments.

In a two-period setting, selling and leasing are identical for new adopters in period 2. This means that if selling and leasing were adopted simultaneously, there must be period 1 adopters who choose the leasing model. Otherwise, the hybrid model is replaced by the selling model, thus not optimal. This in turn gives rise to the following two cases:

Case 1: All period 1 adopters who choose the leasing model continue to lease in period 2, which implies that leasing in both periods is favored over buying in period 1, i.e., $r_1 + r_2 < p_1$. In turn, this means that no consumer would buy in period 1. Then the hybrid model can be fully replaced by the leasing model, thus not optimal. Contradiction.

Case 2: Not all period 1 adopters who choose the leasing model will continue to lease in period 2, i.e., there are some consumers who adopt only in period 1 via leasing. According to Lemma A2, this means that there are no consumers who would adopt only in period 2. Again, the hybrid model is replaced by the leasing model, thus not optimal. Contradiction. □