Doing Less to Do More? Optimal Service Portfolio of Non-profits That Serve Distressed Individuals

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Abstract. Problem definition: Many nonprofit organizations (NPOs) serve distressed individuals who seek relief from hardships such as domestic abuse or homelessness. These NPOs aim to maximize social impact by allocating their limited amount of resources to various activities. Academic/practical relevance: NPOs that serve distressed individuals face a complex task because their clients are often unable to articulate their specific needs. As a result, NPOs are driven to not only offer a variety of services to fulfill different needs, but also engage in advisory activities to minimize mismatches between services clients receive and their true needs. Methodology: We develop a model to study an NPO’s service portfolio and effort allocation decisions under resource constraint. Clients’ progress from distress to resolution is stochastic and depends on the NPO’s efforts in different stages of the service offering. Results: We show that it is optimal for resource-constrained NPOs to offer fewer services and invest more in advisory activities when different types of clients are not evenly mixed in the population, when delays in achieving resolution can significantly blunt the social impact created, when the loss of impact due to not serving a fraction of clients is low, or when there is a limited amount of earmarked funds. Otherwise, it is optimal for NPOs to diversify their service offerings and invest less in advisory activities. Managerial implications: Many NPOs are drawn to maximize the number of clients they serve by increasing the number of services they offer. However, we show that, depending on the characteristics of clients and services, NPOs might be able to generate higher social impact by prioritizing the speed of resolution rather than focusing on the number of clients who achieve resolution. We also present a practical application of our model in the context of domestic abuse.

1. Introduction

Nonprofit organizations (NPOs) that support and serve distressed individuals are often the last resort for those who are seeking relief from hardships such as domestic abuse or homelessness. These societal issues have large economic repercussions. For instance, victims of domestic violence lose a total of 8 million days of work each year in the United States, which amounts to a productivity loss of $8.3 billion per year (Rothman et al. 2007); they are also more vulnerable to depression, suicidal behavior, and HIV infection (World Health Organization 2013). Similarly, homelessness places a significant burden on society in the form of costs associated with shelter, food, mental, and physical health services, which are estimated to cost $35,000 per year per person (Flaming et al. 2009). Higher rates of crime associated with homelessness add to this tally; for instance, in 2010 alone, the state of Georgia spent $300 million in incarcerating homeless individuals (Henrichson and Delaney 2012). There are over 5,000 NPOs in the United States that provide services related to mental health and crisis intervention, civil rights and advocacy, and employment search and training (National Center for Charitable Statistics 2019).

These mission-driven NPOs provide care, education, and shelter, typically through significant personal interaction with their clients (Hasenfeld 2009). In doing so, they face a complex combination of challenges: First, because their clients often vary greatly in terms of their needs (Drucker 1995, Hasenfeld 2009), NPOs might be drawn to offer a variety of services that enable different pathways to wellness (Sawhill and Williamson 2001, Ebrahim and Rangan 2014). Second, because these NPOs are not revenue-generating and rely on external funding from government and private donors, they operate under a scarcity of resources (Feng and Shankikumar 2016). Finally, their clients are often unable to articulate their needs as they are unaware of the...
true causes of their situation (Holdsworth and Tiyce 2013) or have endured traumatic experiences resulting in symptoms of PTSD, low self-esteem, or anxiety (Stewart et al. 2004). As such, clients may seek and receive services that are not best-suited to their needs. Although mismatched clients continue to consume resources, an NPO’s efforts to serve them produce limited social impact. As a result, many NPOs in this domain serve in an interpretive role by providing advisory support to their clients to help them receive the most appropriate services (Emanuel and Emanuel 1992).

Despite the similarity between the challenges faced by these NPOs, they may take different approaches in designing and choosing the portfolio of services to offer. Consider these illustrative examples:1 Daya is a Houston-based NPO that empowers South Asian women who encounter domestic violence (see Daya Houston 2019 and Section 5.2 for additional details). Daya offers a multitude of services ranging from counseling to legal support to fulfill the needs of different types of clients. Daya also administers an intake process, where staff members provide guidance to help each new client to choose the appropriate service type. Similarly, consider First Step, an Atlanta-based NPO, whose goal is to connect homeless individuals to sustainable income (First Step 2019). First Step accomplishes this by offering two types of services: (i) job training and placement for those homeless individuals who can work, and (ii) filing disability insurance claims for those who are unable to work. To guide clients toward the best-suited service, First Step conducts an extended interview covering behavioral and work history related questions. In spite of these advisory efforts, the disability claims of roughly 20% are rejected, and a similar fraction of clients who are being trained and placed in jobs are unable to fulfill their job obligations.

In contrast to Daya and First Step that offer different services for different client types, some NPOs focus on offering a smaller subset of services. A case in point is Georgia Works (GW), an NPO in Atlanta, Georgia that aims to transform chronically homeless men into self-sufficient members of society (Georgia Works 2019). Although GW could offer both disability assistance and employment services to homeless individuals, it deliberately offers only employment services, but not disability assistance.2 GW spends around 12% of its overall resources on the intake process to help clients understand program requirements. This approach has significantly reduced the number of mismatches to the extent that nearly 70% of clients who join GW can complete the arduous requirements of the program.

Motivated by the different approaches taken by these NPOs, we focus on the service and portfolio design of NPOs that serve distressed individuals. Despite growing evidence documenting challenges faced by such NPOs, their operational issues have received limited attention from the academic community (Berenguer and Shen 2019, Besiou and Van Wassenhove 2020). Many practically important questions remain unanswered, which we address in this paper. For NPOs that serve distressed individuals, (i) what is the optimal portfolio of services that maximizes social impact? (ii) what are the optimal investments in advisory and service delivery activities that maximize social impact? and (iii) how are these decisions affected by the focus on maximizing social impact and presence of earmarked funding?

To answer the those questions, we propose an analytical model, in which an NPO that has a limited amount of resources aims to maximize the social impact generated by serving its clients. The social impact depends both on the portfolio of services offered and on the speed at which clients obtain resolution for their needs. The NPO can hasten clients’ progress (from distress to resolution) by allocating more resources to service delivery efforts. Clients have heterogeneous needs, and the NPO can also reduce the occurrence of mismatches between clients’ needs and services they receive by allocating resources to advisory effort. We characterize the optimal service portfolio of NPOs, and generate insights on when and why an impact-focused NPO can generate higher social impact than an output-focused NPO (that maximizes the number of clients who reach resolution). We also examine the effect of earmarked funds on an NPO’s optimal service portfolio and its resulting social impact.

Our analysis yields several insights of managerial consequence. Although output-focused NPOs provide a variety of services to maximize the number of clients they serve, we show that an impact-focused NPO could generate higher social impact by providing fewer services and investing more in advisory activities. This is especially true when different types of clients are not evenly mixed in the population, when delays in achieving resolution can blunt the social impact created, when the loss of impact due to not serving a fraction of clients is low, and when the amount of earmarked funds is small. Otherwise, it is optimal for the NPO to diversify its service offerings and invest less in advisory activities. We present these insights in a practical setting based on the context of domestic abuse. We also further generalize our results in several directions by capturing situations where some clients require more than one type of service, some clients have more complex needs than others, and some clients drop out from service delivery.

The remainder of the paper is organized as follows. We review the related literature in Section 2, and present the model in Section 3. Our main results are presented in Section 4. We provide an illustrative case study based on the context of domestic abuse in Section 5, and present model extensions in Section 6. We conclude with a summary of managerial insights and directions for future research in Section 7. Proofs of the main analytical results are presented in the online appendix, and other technical details are available in an electronic companion.
2. Literature Review

A common objective shared by many nonprofit organizations is to maximize the social impact generated by their activities (Kramer 1981, Kalkanci et al. 2019, Cachon et al. 2020). However, the limited and uncertain availability of funds forces NPOs to manage challenging trade-offs in their operations (Devalkar et al. 2017). The nature of those trade-offs varies depending on whether NPOs provide emergency relief or address chronic societal problems (Feng and Shanthikumar 2017). In this paper, we focus on NPOs that offer products (e.g., food, medical supplies, etc.) and services (e.g., legal aid, job training, and counseling) to beneficiaries. A stream of research has focused on the quality-adjusted coverage of health, education, and child adoption (Slaugh et al. 2016, Virudachalam et al. 2018, Mehrotra and Natarajan 2020). For instance, Mehrotra and Natarajan (2020) propose incentive mechanisms that the humanitarian organization could offer to increase the number of clients that seek health services. Similarly, Virudachalam et al. (2018) study the performance of nonprofit education systems and propose monetary incentives to motivate teachers to improve students’ performance on tests. In this paper, we focus on serving distressed individuals, where the objective of the NPO is to help clients progress from distress to resolution promptly. We formalize this impact-focused objective and show how NPOs can maximize that by choosing the portfolio of services offered and by investing in advisory support and delivery of each of those services. However, because these organizations have a limited amount of funds that can be used for various service delivery activities, they often face a resource allocation dilemma (Berenguer and Shen 2019). de Véricourt and Lobo (2009) examine how an NPO can allocate resources between revenue-generating and economically challenged clients, who sort themselves into separate services. Similarly, Kotsi et al. (2017) study how NPOs should allocate their limited funds between program, fundraising, and administration. These studies, however, do not account for issues such as the loss of social impact due to mismatches and delays in clients obtaining resolution. We complement these studies by considering that distressed clients are often unable to identify services that suit their needs, which can lead to mismatches. Although mismatched clients continue to consume resources, NPO’s efforts to serve them produce limited social impact. This leads to service design problem for these NPOs.

Service providers can improve the quality and duration of their services by optimally allocating resources among service stages (Karmarkar and Pitbladdo 1995, Roth and Menor 2003). A group of studies has focused on effort allocation toward improving the experience of customers in contexts where service stages are independent (Soteriou and Chase 2000, Tong et al. 2016) or interdependent (Bellos and Kavadias 2021). Although we also consider interdependent service stages (i.e., advisory and service delivery efforts), the allocation of resources is subject to constraints (which can also be earmarked). Another group of studies has focused on service contexts in which the time that a provider spends with a customer is a key determinant of service value, but a higher time spent in service delivery leads to longer delays for customers waiting to receive services (Debo et al. 2008, Anand et al. 2011, Tong and Rajagopalan 2014). These studies consider a single type of service to serve the needs of a single type of customer; thus, the issue of mismatch between clients’ needs and
services offered does not arise. In contrast, we consider heterogeneity in clients’ needs and capture situations where mismatches arise and lead to loss of value and delays in service delivery.

Considering the effect of mismatches on delaying service delivery, Shumsky and Pinker (2003) and Lee et al. (2012) study the design of service processes with a front line of gatekeepers who may refer the customer to a specialist (e.g., in call centers and hospitals). They show that the provider can reduce delay and mistreatment costs by properly designing incentive schemes. Similarly, Alizamir et al. (2013) study the trade-off between improving accuracy (requiring additional tests) and delaying the provision of service to other customers. Although they consider the issue of service delays in the diagnostic stage, they assume that, once clients are assigned to a service, the outcomes of the service delivery are known (with mismatches generating less value). We complement these studies by considering situations where clients’ paths to resolution and outcomes of service delivery stages are also stochastic. Specifically, we consider that clients may move between service delivery stages or even exit the system before (or without) reaching resolution. Accordingly, we use a discrete-time Markov Chain model to capture the client’s progress, whereas some of the above papers have used a queuing model.

Finally, we study the optimal portfolio of services that the NPO should offer, a question that has not been studied in the above papers. We also consider that in addition to initial mismatches, service delivery stages are also subject to uncertainty as clients may identify mismatches while in the system and switch in between service delivery stages. The combination of these effects allows us to capture salient trade-offs in the context of providing services to distressed clients. Specifically, in choosing the optimal portfolio of services, we capture a trade-off between the speed of resolution and quantity of clients served. Further, in determining the optimal effort allocation among service stages, we capture a trade-off between improving matching for incoming clients (via advising) and accelerating resolution for matched clients (through service delivery). We discuss these in more detail in the next section.

3. Model

In this section, we propose a model of service and portfolio design of an NPO that serves distressed clients with different needs. The NPO has limited financial resources in each period, denoted by $S > 0$, and has to allocate that to different service stages. In order to capture the heterogeneity in clients’ needs, we focus our main analysis on a setting with two client types, denoted by $i \in \{a, b\}$, and two service types, denoted by $j \in \{A, B\}$. The $A$-type ($B$-type) service is best suited to the needs of $a$-type ($b$-type) clients. We later extend our model and analysis to a setting with multiple client types and service types (in Section 5). We denote by $\gamma \in (0,1)$ the proportion of $a$-type clients and by $1 - \gamma$ the proportion of $b$-type clients. Without loss of generality, we normalize the NPO’s total demand in each period to one.

Clients’ path from distress to resolution is as follows: Clients enter the NPO through an intake process, wherein the NPO provides advisory support to help clients choose among its portfolio of services. After a client chooses a service, the NPO provides that service immediately and on a regular basis until either the client’s need is resolved or the client identifies a mismatch and seeks a different service. Following the intake process, clients may seek and receive services that are not best-suited to their needs; clients may not identify mismatches immediately and continue receiving incorrect services; and even when a client identifies the mismatch and starts receiving their best-suited service, their problem may not be resolved immediately because of the inherent complexity of their needs. To model the client’s transition process with stochastic movements between states, we employ a discrete-time Markov chain model, which is commonly used in operations management to model various service delivery systems (e.g., Alagoz et al. 2004, Rahmani et al. 2017; also see details in the e-companion). Although we assume that transition probabilities are stationary in the majority of our discussions, we later show that the key insights are robust to situation where transition probabilities are time- or state-dependent (see Section 6.2 and Section 6.4).

3.1. States and Transition Probabilities

In each period, a client can be in one of the following four states: (i) State $G$, where the client has entered the system and is choosing a service to receive, (ii) State $A$, which indicates the client is receiving the $A$-type service, (iii) State $B$, which indicates the client is receiving the $B$-type service, and (iv) State $R$, which indicates the client has obtained resolution for their need. Figure 1 illustrates the state transitions for different client types when the NPO offers both types of services.

3.1.1. Transitioning from State $G$ to State $A$ or $B$. We denote the NPO’s advisory effort per client by $\epsilon_G \geq 0$. This effort typically involves explanation of services offered, extended in-take interviews, professional tests of skills, or administering psychological and behavioral examinations. We define $\theta (\epsilon_G)$ as the probability of a matched transition, that is, when an $a$-type ($b$-type) client transitions from state $G$ to state $A$ (state $B$). We consider $\theta (\epsilon_G) = \theta + \theta_0 \cdot \epsilon_G$, where parameter $\theta_0 > 0$ represents the rate at which a unit of advisory effort
increases the probability of matched transition. The parameter $\theta \geq 0$ denotes the probability of a matched transition when the NPO offers no or only a basic guideline about the services it offers (i.e., $c_G = 0$). For instance, even when the NPO does not provide personal guidance, clients may be able to recognize the best-suited service to their needs through the content available in the NPO’s brochure, website, or mobile application. Throughout our analysis, without loss of generality, we consider combinations of parameters that ensure the probability function $\theta(e_G)$ lies in the unit interval (see details in the e-companion).

### 3.1.2. Transitioning from State $A$ or $B$ to State $R$

In each period, the NPO exerts service delivery effort $e_j \geq 0$ to serve each client who is in state $j$ for $j \in \{A, B\}$. These service delivery efforts could be in the form of assisting with legal procedures, filing disability claims, or financial recovery planning. We define $\mu(e_j)$ as the transition probability of an $a$-type client from state $A$ to state $R$, and $\mu(e_j)$ as the transition probability of $b$-type client from state $B$ to state $R$. Accordingly, $1 - \mu(e_A)$ and $1 - \mu(e_B)$ denote probabilities that the client’s need is not resolved in a given period, even though they have received the best-suited service delivery; hence, the client stays in that same state for another round of service delivery. This implies that for a given effort level $e_j$, the sojourn time of state $j$ follows a geometric distribution. We consider $\mu(e_j) = \mu_j \cdot e_j$ for $j \in \{A, B\}$, where parameter $\mu_j > 0$ denotes the transition rate of $j$-type service per unit of service delivery effort. To simplify the exposition, in the main analysis, we consider similar transition rates for both services (i.e., $\mu_A = \mu_B = \mu$). Note that transition probabilities $\mu(e_j)$ depend on efforts, which can vary between the two services even when transition rates are the same. In addition, without loss of generality, we consider combinations of parameters that ensure the probability function $\mu(e_j)$ for $j \in \{A, B\}$ lies in the unit interval (see details in the e-companion). We later extend our analysis in several directions and show that the overall insights continue to hold in situations where transition rates are also different across the two services or when they are state- or time-dependent (in Section 6.2 and Section 6.4).

### 3.1.3. Transitioning from State $A$ ($B$) to State $B$ ($A$)

As explained, clients may transition from state $G$ to a service delivery state $A$ or $B$ that is not best suited to their needs. We refer to this as a mismatch (i.e., when $a$-type ($b$-type) client transitions from state $G$ to state $B$ (state $A$)). For instance, in the case of Daya, mismatches arise when clients who need counseling for psychological abuse instead seek and receive legal representation for filing a divorce. In the case of mismatches, it takes, on average, a longer amount of time (more periods) for clients to reach the resolution state. In each period, the client identifies the mismatch with the probability $1 - \alpha \in (0, 1)$. Subsequently, they can begin to receive the appropriate service that can take them toward resolution (state $R$). For instance, a woman who seeks a legal service for filing a divorce may subsequently realize that this is not a suitable service for her (e.g., due to the attendant loss of child custody), and therefore might switch to counseling and job training services. In this sense, $\alpha$ captures the likelihood of the client remaining in a mismatched state for another round of
service delivery. We also generalize this model to situations where clients may drop out from service delivery (in Section 6.3).

### 3.2. NPO’s Effort Allocation Problem

In formalizing the NPO’s optimization problem, we utilize academic and practitioner reports to identify appropriate measures for the objective function and constraints (Sawhill and Williamson 2001, GECES 2013).

#### 3.2.1. Objective Function

The NPO’s objective is to maximize the social impact generated by helping clients reach their long-term resolution; however, it is also important to achieve resolution speedily as there are socioeconomic costs associated with clients being in distress (Gerberding et al. 2003, Oliver et al. 2019).

We denote the social value the NPO generates by helping each client reach their long-term resolution by \( I \geq 0 \), which we refer to as the *impact factor*. For instance, in the context of Georgia Works, which focuses on homelessness, social impact is created when an individual becomes self-sufficient through rehabilitation and gainful employment. The social impact generated when a client is free of distress is often independent of the root cause of their problem; without loss of generality, we consider the same impact factor for both client types.

In addition, the NPO accounts for the loss of social impact due to delays in achieving resolution, which we capture through \( w_i \cdot T_i (\cdot) \). The parameters \( w_i \geq 0 \) for \( i \in \{a, b\} \), which we refer to as the *delay costs*, captures the loss of social impact per period a client remains in distress before reaching the resolution state \( R \). The function \( T_i (\cdot) \) determines the expected time (i.e., number of periods) that \( i \)-type clients spend in the system before reaching the resolution state. To simplify exposition, in the main analysis, we consider similar delay costs for both client types. We later show the robustness of the key insights to situations where delay costs vary between different client types (in Section 6.4).

Based on the transition probabilities of the Markov Chain (illustrated in Figure 1), we find that for given advisory and service delivery efforts, it takes an \( a \)-type client on average \( T_a (e_G, e_A) \) periods to reach state \( R \), such that

\[
T_a (e_G, e_A) = 1 + \theta (e_G) \left( \frac{1}{\mu (e_A)} \right) + (1 - \theta (e_G)) \left( \frac{1}{1 - \alpha} + \frac{1}{\mu (e_A)} \right).
\]

(1)

Similarly, for a \( b \)-type client, it takes on average \( T_b (e_G, e_B) \) periods to reach state \( R \) (see details in the e-companion).

As shown in Equation (1), it takes longer for mismatched clients to reach the resolution state. Also, note that the average number of periods to reach state \( R \) decreases as the NPO increases its advisory and service delivery efforts.

Accordingly, we define the total expected social impact \( \text{TEI} \) that the NPO generates when it offers both types of services as follows:

\[
\text{TEI} (e_G, e_A, e_B) = \gamma (1 - w_a \cdot T_a (e_G, e_A)) + (1 - \gamma) (1 - w_b \cdot T_b (e_G, e_B)).
\]

(2)

#### 3.2.2. Resource Constraints

By considering the steady-state distribution of clients in various stages of the service system, we obtain the average number of clients in state \( A \) in each period as follows:

\[
C_A (e_G, e_A) = \gamma \left( \frac{\theta (e_G)}{\mu (e_A)} + \frac{1 - \theta (e_G)}{\mu (e_A)} \right) \frac{1}{1 - \alpha}.
\]

(3)

Similarly, the average number of clients in state \( B \) in each period is

\[
C_B (e_G, e_B) = (1 - \gamma) \left( \frac{\theta (e_G)}{\mu (e_B)} + \frac{1 - \theta (e_G)}{\mu (e_B)} \right) \frac{1}{1 - \alpha} + \gamma \left( \frac{1 - \theta (e_G)}{1 - \alpha} \right)
\]

(4)

By considering the steady-state distribution of clients in various stages of the service system, we obtain the average number of clients in state \( A \) in each period as follows:

\[
C_A (e_G, e_A) = \gamma \left( \frac{\theta (e_G)}{\mu (e_A)} + \frac{1 - \theta (e_G)}{\mu (e_A)} \right) \frac{1}{1 - \alpha}.
\]

(3)

Similarly, the average number of clients in state \( B \) in each period is

\[
C_B (e_G, e_B) = (1 - \gamma) \left( \frac{\theta (e_G)}{\mu (e_B)} + \frac{1 - \theta (e_G)}{\mu (e_B)} \right) \frac{1}{1 - \alpha} + \gamma \left( \frac{1 - \theta (e_G)}{1 - \alpha} \right)
\]

(4)

#### 3.2.3. NPO’s Effort Allocation Problem When Offering Both Types of Services

Based on the NPO’s objective function (in Equation (2)) and its resource constraint (in Equation (4)), the NPO chooses its optimal advisory and service delivery efforts, denoted by \((e_{G}, e_{A}, e_{B})\), by solving the following optimization problem:

\[
\max_{(e_{G}, e_{A}, e_{B})} \text{TEI} (e_{G}, e_{A}, e_{B})
\]

s.t. \( e_{G} + C_{A} (e_{G}, e_{A}) \cdot e_{A} + C_{B} (e_{G}, e_{B}) \cdot e_{B} \leq S \)

(4)

The above optimization problem has an objective function that is jointly concave in the decision variables. In addition, it highlights the interdependence between advisory and service delivery efforts. Although increasing advisory effort \( e_{G} \) increases the likelihood of a match (i.e., higher transition probability from state \( G \) to a matched state \( A \) or \( B \)), it comes at the cost of limiting the NPO’s service delivery efforts \( e_{A}, e_{B} \).

#### 3.2.4. NPO’s Effort Allocation Problem When Offering One Type of Service

When an NPO offers only one type of service, clients who do not choose that service may (temporarily) exit the system. We denote the exit
state by \( E \). We assume that clients who exit the system due to a mismatch can still recognize that the service offered by the NPO is indeed appropriate for them and return to receive the service. Figure 2(a) illustrates the states and transition probabilities of \( a \)-type clients when the NPO offers only the \( A \)-type service (note that the model for offering only the \( B \)-type service is analogous).

After receiving the advisory support, an \( a \)-type client transitions to the \( A \)-type service with probability \( \theta(e_{C}) \), or transitions to state \( E \) with probability \( 1 - \theta(e_{C}) \). The mismatched \( a \)-type clients return to the system with the probability \( 1 - l \alpha \in (0, 1) \) with \( l \in (1, 1/\alpha) \). The parameter \( l \), which we refer to as the latency factor, captures the additional difficulty for clients to recognize a mismatch after they exit the system. Given that such \( a \)-type clients return to the system only when they discover that the \( A \)-type service is the best-suited service to their needs, they directly transition to state \( A \) and receive services until they transition to the resolution state. We also show the robustness of the key insights to situations where clients who exited the system may not return (in Section 6.3).

Figure 2(b) illustrates the states and transition probabilities of \( b \)-type clients when the NPO offers only the \( A \)-type service. After receiving the advisory support, a \( b \)-type client transitions to the mismatched state \( A \) with probability \( 1 - \theta(e_{C}) \), and exits the system without receiving further service from the NPO with probability \( \theta(e_{C}) \). Hence, for \( b \)-type clients, the absorbing state is state \( E \). In such cases, the NPO incurs a negative social impact \( pl^{-} \) due to not helping \( b \)-type clients reach resolution. We refer to parameter \( p \geq 0 \) as the loss of impact factor. Further, we consider the NPO’s loss of impact due to delays experienced by \( b \)-type clients before transitioning to state \( E \).

Accordingly, the objective function and resource constraint of the NPO that offers only the \( A \)-type service can be obtained as follows:

\[
\max_{\{e_{G}, e_{A}\}} \text{TEI}_{A}(e_{G}, e_{A}) \triangleq \gamma(1 - w_{A} T_{d}(e_{G}, e_{A})) + (1 - \gamma) \left( -pl^{-} - w_{l} \left( 1 + \frac{1 - \theta(e_{G})}{1 - \alpha} \right) \right)
\]

\[
s.t., e_{G} + C_{A}(e_{G}, e_{A}) - e_{A} \leq S, \quad e_{G} \geq 0, e_{A} \geq 0,
\]

where \( C_{A}(e_{G}, e_{A}) \) is as defined in Equation (3), and

\[
T_{d}(e_{G}, e_{A}) \triangleq \begin{cases} 
1 + \theta(e_{G}) \left( \frac{1}{\mu(e_{A})} \right) & \text{in a case of match} \\
(1 - \theta(e_{G})) \left( \frac{1}{1 - l \alpha} + \frac{1}{\mu(e_{A})} \right) & \text{in a case of mismatch}
\end{cases}
\]

Note that it takes longer for \( a \)-type clients who exited the system to identify their mismatch compared with \( b \)-type clients who remain in the system (i.e., \( 1/l \alpha \geq 1/(1 - \alpha) \) for \( l \in (1, 1/\alpha) \)). The NPO’s optimization problem when it offers only \( B \)-type service is analogous and presented in the e-companion. We denote the NPO’s optimal efforts when it offers only the \( A \)-type service by \( \{e^{r}_{G}, e^{r}_{A}\} \), and when it offers only the \( B \)-type service by \( \{e^{r}_{G}, e^{r}_{B}\} \).

### 3.3. NPO’s Portfolio Design Problem

Considering the optimal advisory and service delivery efforts under different service offerings, the NPO decides which combination of services to offer to maximize the social impact it can generate. The optimal portfolio of services is obtained by solving the following problem:

\[
\text{TEI}^{\star} = \max \{ \text{TEI}_{A}(e^{r}_{G}, e^{r}_{A}), \text{TEI}_{A}(e^{r}_{G}, e^{r}_{B}), \text{TEI}_{B}(e^{r*}_{G}, e^{r*}_{B}) \}
\]

\[
s.t. \ (e^{r}_{G}, e^{r}_{A}, e^{r}_{B}); \text{are optimal efforts when the NPO offers both types of services, (e^{r*}_{G}, e^{r*}_{A}); are optimal efforts when the NPO offers only A-type service, (e^{r*}_{G}, e^{r*}_{B}); are optimal efforts when the NPO offers only B-type.}
\]

In choosing the optimal portfolio of services, the NPO has to balance the trade-off between three factors. Offering more services helps the NPO serve more clients and potentially generate higher social impact (I). At the same time, offering many services could reduce service delivery efforts (due to limited resources), which results in loss of impact due to delayed resolution (\( w \cdot T_{i}(. \))). Reducing the number of services could also lead to loss of social impact as some clients never receive resolution (\( pl^{-} \)). As a result, the optimal efforts and service portfolio decisions are intertwined. We characterize these decisions in the next section. The parameters and notations used in this paper are summarized in Table A-1 in the online appendix.
4. Optimal Design and Portfolio of Services

In this section, we answer our research questions by solving the NPO’s effort allocation and portfolio design problems. To simplify the exposition, we present the closed-form characterization of the NPO’s optimal efforts in the e-companion. Replacing the optimal efforts in the total expected impact function for different service offerings (i.e., both, A-only and B-only) and comparing the resulting impacts, the next proposition characterizes the NPO’s optimal portfolio of services.

Without loss of generality, we focus on scenarios where $\gamma \geq 1/2$ (the result for $\gamma < 1/2$ is analogous).

**Proposition 1. (Optimal Portfolio of Services).** Suppose $\gamma \geq \frac{1}{2}$. Then, there exist two thresholds $\Phi_1$ and $\Phi_2$ (with $\Phi_2 \geq \Phi_1$) such that it is optimal for the NPO to offer:

i. both types of service when $\frac{1}{\gamma} \geq \Phi_2$,

ii. only A-type service when $\Phi_1 \leq \frac{1}{\gamma} < \Phi_2$, and

iii. only B-type service when $\frac{1}{\gamma} < \Phi_1$.

The thresholds $\Phi_1$ and $\Phi_2$ are decreasing in $\rho$ and in $\gamma$.

Proposition 1 shows that the NPO’s choice between offering both types of services, only A-type service, and only B-type service depends on the social impact generated relative to the cost of delayed resolution ($1/\gamma$), which we refer to as the relative impact. When the relative impact is high, delays do not significantly blunt social impact of resolution. In this scenario, the NPO should offer both types of services to help all clients reach resolution. However, when the relative impact is low, it is optimal for the NPO to focus on the delivery of only one type of service. Proposition 1 also shows that diversifying the service offerings is optimal when the NPO incurs a high loss of impact from not serving a fraction of clients (i.e., $\rho$ is high) or when the likelihood that clients who exited the system identify their mismatch is low (i.e., $\gamma$ is high).

Figure 3 illustrates the NPO’s optimal service portfolio with respect to the delay cost ($\omega$) and the mix of clients ($\gamma$) for different values of the loss of impact factor ($\rho$). As the delay cost increases, it becomes optimal for the NPO to focus on delivering only one type of service. When the NPO focuses on one type of service, it limits the number of clients it serves; however, those clients who are served receive resolution to their needs promptly. Indeed, this is similar to the approach that Georgia Works has taken, by focusing on providing only employment services to homeless individuals. Figure 3 also shows that the region in which it is optimal to offer both types of services expands as $\rho$ increases. The reason is that, when $\rho$ is high, providing even a small amount of resources to each client is better than not serving them at all. For instance, this could occur when clients do not have access to other resources or organizations (e.g., in rural areas). An unintended consequence of diversifying services is that the NPO can only allocate a limited amount of resources to advisory effort, which could potentially lead to more mismatches. The next proposition formalizes this result. Recall that $(e_G^{*}, e_A^{*}, e_B^{*})$ denote the optimal efforts when both services are offered, $(e_G^{**}, e_A^{**}, e_B^{**})$ denote the optimal efforts when only the A-type service is offered, and $(e_G^{***}, e_A^{***}, e_B^{***})$ denote the optimal efforts when only the B-type service is offered.

**Proposition 2. (Optimal Efforts).** Comparing NPO’s optimal efforts when it offers both services, only A-type service, and only B-type service,

i. $e_G^{*} \leq \max \{e_G^{**}, e_G^{***}\}$, $e_A^{*} \leq e_A^{**}$, and $e_B^{*} \leq e_B^{**}$.

ii. $e_G^{**} \leq e_G^{***}$, $e_A^{**} \geq e_A^{***}$, and $e_B^{**} \geq e_B^{***}$ if and only if $\gamma \geq 1/2$.

iii. $e_G^{*}/e_A^{*}$, $e_G^{**}/e_B^{*}$, $e_G^{***}/e_B^{*}$, $e_A^{**}/e_A^{***}$, $e_B^{**}/e_B^{***}$ are nondecreasing in $\mu$ and in $\alpha$.

iv. $e_G^{*}$, $e_G^{**}$, and $e_G^{***}$ are nondecreasing in $S$, and $e_A^{*}$, $e_B^{*}$, $e_A^{**}$, and $e_B^{**}$ are increasing in $S$.

Proposition 2 shows that it is optimal for the NPO to allocate more resources to advisory effort when it offers a single type of service. As shown in Proposition 1, offering a single type of service is optimal when the relative impact factor ($1/\omega$) or the loss of impact factor ($\rho$) is small. Combining these results implies that the optimal advisory effort is nonincreasing in $1/\omega$ and $\rho$. The reason is that when the impact gain (loss) due to (not) serving a fraction of clients is high (i.e., $1/\omega$ or $\rho$ is high), the NPO should give priority to offering more services rather than reducing mismatches. Note that within each portfolio strategy (offering one service or both services), the NPO’s optimal effort allocation does not change with $\rho$ and $I$. However, the optimal effort levels do depend on $\rho$ and $I$ once the optimal service portfolio decisions are internalized.

The proposition also shows that the NPO should invest more in delivering the service that is best suited to the majority of clients. In addition, the NPO should invest more in advisory effort relative to service delivery efforts when the transition rate ($\mu$) or the probability of clients remaining in the mismatch state ($\alpha$) is high. When $\mu$ is high, due to the faster progression of clients to resolution, fewer clients remain in the service delivery states. As a result, the overall amount of service delivery resources consumed is lower, allowing the NPO to allocate more resources to advisory effort relative to the service delivery efforts. When $\alpha$ is high, there is a larger number of mismatched clients in the system. This makes it optimal for the NPO to exert higher advisory effort relative to service delivery effort. Finally, the last part of the proposition shows that an increase in the availability of resources generally results in greater efforts across the board.

Figure 4 illustrates how the NPO’s service portfolio and efforts vary with available resources ($S$), the match probability with basic guidance ($\Theta$), and the latency factor ($l$). Note that increases in $S$ and $\Theta$ are socially
beneficial as they make more resources available for the NPO (directly in the case of $S$ and indirectly by reducing mismatches when $\theta$ is higher). The NPO is able to apply these additional resources to increase the effort levels for each of the clients who are in the system. When a substantial amount of resources are freed up (i.e., when $S$ or $\theta$ is sufficiently high), the NPO is able to generate a higher social impact by providing both types of services. As a result, the optimal effort levels of the NPO exhibit nonmonotonic responses to increases in $S$ and $\theta$. Unlike $S$ and $\theta$, an increase in $l$ is not socially beneficial; that is, a larger $l$ implies that clients who leave the NPO due to mismatches face an even longer path to resolution. When the latency factor is high, it is optimal to ensure that none of the clients exit the system due to a mismatch. In such cases, the NPO offers both types of services to eliminate any role latency might play in delaying resolution. At this point where the NPO switches from offering only $A$-type service to offering both services, we observe a drop in the effort levels $e_G$ and $e_A$ so that resources may be allocated to provide both services.

In summary, the analyses in this section suggest that it is optimal for resource-constrained NPOs to offer fewer services and invest more in advisory activities when different types of clients are not evenly mixed in the population, when delays in achieving resolution can significantly blunt the social impact created, when the loss of impact due to not serving a proportion of clients is low, or when the likelihood that clients who exited the system identify their mismatch is high. Otherwise, it is optimal for NPOs to diversify their service offerings and invest less in advisory activities.

4.1. Comparison of Impact-Focused and Output-Focused NPOs

Although NPOs are encouraged to measure impact by using quantifiable indicators (which vary depending

Figure 3. (Color online) Optimal Portfolio of Services

Note. Parameters: $\theta = 0.5; \theta_o = 0.01; \mu = 0.03; \alpha = 0.65; l = 75; l = 1; S = 55.$

Figure 4. (Color online) Optimal Service Portfolio and Efforts

Note. Parameters are the same as in Figure 3 with $w = 13, \gamma = 0.6$, and $\rho = 0.$
on the context), in practice, many NPOs focus on output instead of impact (Ebrahim and Rangan 2014). The European Venture Philanthropy Association (EVPA) defines output as the tangible result of the NPO’s activities (e.g., number of people who are served), and defines impact as changes and benefits resulting from the NPO’s activities (e.g., effects on the target population and society). Based on a survey of NPOs conducted by the EVPA, the objective measurements were based on output in the majority of cases (84%); but the survey also revealed an increase in the percentage of organizations attempting to measure changes and benefits based on impact (EVPA 2013). As reported by Sawhill and Williamson (2001, p. 1), “[output] metrics are certainly important, but they don’t measure the real success of an organization in achieving its mission.”

In this section, we consider an output-focused NPO as a benchmark to understand the importance of impact maximization. In the context of nonprofits that serve distressed individuals, an NPO can measure its output by counting the number of clients that reach the resolution state in each period. Although output-focused NPOs do not measure impact, they can still generate social impact from their services. However, their social impact may be limited due to their quest to allocate resources to maximize their social impact (Sawhill and Williamson 2001). We find that multiple combinations of efforts could lead to the same output. That is because (i) output-focused NPOs do not internalize the effectiveness of services (i.e., the time taken by clients to achieve long-term resolution), and (ii) all clients who reach their best-suited service delivery state eventually proceed to the resolution state (i.e., they do not drop out from the system).

For the sake of comparison, in Proposition 3, we present a conservative estimate of the additional impact that NPOs can generate by being impact-focused rather than output-focused. Accordingly, among the multiple solutions for the output-focused NPO’s problem, we consider the solution that yields the highest social impact. We denote the maximum total expected impact that an output-focused NPO can generate by TEI^O, and its best possible efforts allocation by (e^0_C, e^0_A, e^0_B). We denote the total expected impact and optimal efforts of an impact-focused NPO by TEI and (e^*_C, e^*_A, e^*_B), respectively.

**Proposition 3.** (Impact-Focused vs. Output-Focused NPOs). *It is optimal for an output-focused NPO to offer both types of services. Consider Φ_1 and Φ_2 defined in Proposition 1. Then,*

i. When \( \frac{1}{w} \geq \Phi_2, \text{TEI} > \text{TEI}^O, e^*_C = e^0_C, e^*_A = e^0_A \) and \( e^*_B \leq e^0_B \).

ii. When \( \frac{1}{w} \leq \Phi_2, \text{TEI} > \text{TEI}^O, e^*_C \geq e^0_C, e^*_A > e^0_A \) and \( e^*_B \leq e^0_B \).

iii. When \( \frac{1}{w} < \Phi_1, \text{TEI} \geq \text{TEI}^O, e^*_C \geq e^0_C, e^*_A \leq e^0_A \) and \( e^*_B > e^0_B \).

Further, \( \text{TEI} - \text{TEI}^O \) is nondecreasing in \( w \), and nonincreasing in \( ρ \) and in \( l \).

Proposition 3 shows that an output-focused NPO finds it optimal to offer both types of services (see Section 6.3 for exceptions). When an NPO offers a single type of service, only the fraction of clients whose needs suit the service offered will reach resolution. Note that NPOs that serve distressed individuals cannot coerce clients to receive a particular type of service. Thus, by offering both types of services, an output-focused NPO can ensure that all clients who enter the system will progress to resolution, even though it may take them longer to do so. When the relative impact is high (\( l/w \)), an impact-focused NPO also offers both services (Proposition 1). Hence, the two types of NPOs can perform the same in terms of service and portfolio design. In contrast, when the relative impact (\( l/w \)) is low, an impact-focused NPO finds it optimal to offer a single type of service; thus, the two types of NPOs perform differently. Further, when the two NPOs offer different service portfolios, an output-focused NPO would allocate less resources in advisory effort compared with an impact-focused NPO. This is due to the fact that the output-focused NPO offers both services, which restricts the amount of resources available for advisory efforts. These effects also continue to hold in situations where some clients drop out before obtaining resolution to their needs (see details in Section 6.3). Moreover, Proposition 3 shows that the additional impact generated by an impact-focused NPO decreases when the delay cost (\( w \)) is low, the loss of impact from not serving a fraction of clients (\( ρ \)) is high, or the likelihood of identifying mismatches for those clients who exit the system is low (\( l \) is high). The intuition behind these results is that these conditions imply significant losses of impact from not serving a fraction of clients and thus increase the benefits of diversifying service offerings for an impact-focused NPO.

Figure 5(a) illustrates the resultant difference between the impact generated by impact- and output-focused NPOs as functions of the mix of clients (\( γ \)) and for different values of the loss of impact factor (\( ρ \)). In this example, the impact-focused NPO offers only the A-type service, whereas the output-focused NPO offers both services. As shown in Proposition 3, \( \text{TEI} \geq \text{TEI}^O \) for all values of the loss of impact factor (\( ρ \)). In addition, the gap between the total expected impact of the two types of NPOs decreases as \( ρ \) increases. Naturally, the impact-focused NPO can generate a higher social impact when the proportion of a-type clients is higher and not serving b-type clients does not lead to high loss of impact. The output-focused NPO (which
offers both services) can also generate a higher social impact as $\gamma$ increases, while its impact does not change with $\rho$ given that it always offers both types of services. When $\gamma$ is large, output-focused NPO can allocate higher resources to advisory and $A$-type services, and reduce the allocation of its resources to the $B$-type service (Proposition 2). Eventually, as $\gamma \to 1$, even the output-focused NPO suppresses $B$-type service effort such that both types of organizations can generate the same overall impact.

The difference between impact- and output-focused NPOs can also be observed in the speed with which clients obtain resolution. Figure 5(b) shows the cumulative distributions of the time spent by $a$-type clients prior to reaching resolution in impact- and output-focused NPOs. The distribution of resolution times of clients in the output-focused NPO has first-order stochastic dominance over the distribution in the impact-focused NPO. In other words, $a$-type clients spend less time in the impact-focused NPO as it prioritizes speedy resolution of clients’ issues. In Section 5.2, we compare the performance of impact-focused and output-focused NPOs in the context of domestic abuse and report the minimum impact gain and reduction in clients’ expected delays prior to reaching resolution.

4.2. The Effect of Earmarked Funds

NPOs often receive funds that are earmarked for delivering a specific type of service. Depending on the social context, earmarked funds can contribute from 40% to 87% of the overall funding received by large NPOs such as the Red Cross and the World Bank (Strom 2008, Tortora and Steensen 2014, Stauffer et al. 2016). In the context of domestic violence, state agencies often provide funds to NPOs that are earmarked to help victims of specific types of abuse (NACVAW 2001, Massachusetts Health and Human Services 2008).

Several instances of earmarked funding also exist in the context of services provided by NPOs to homeless individuals (Swiatecki 2018, Lee 2020, Myers 2020). In this section, we study the effect of additional earmarked funds on the NPO’s optimal service and portfolio design. Without loss of generality, throughout the paper, we consider the case where the NPO has received additional earmarked funds to invest exclusively in the $B$-type service delivery, denoted by $\xi_B$. Thus, when $\xi_B > 0$, the NPO’s resource constraints can be modified as follows:

$$e_G + C_A (e_G, e_A) \cdot e_A + C_B (e_G, e_B) \cdot e_B \leq S + \xi_B,$$

$$e_G + C_A (e_G, e_A) \cdot e_A \leq S.$$  

The additional constraint (8) implies that the earmarked funds cannot be invested in advisory effort and $A$-type service delivery effort. As presented in Equations (7) and (8), we consider earmarked funds that are in addition to the NPO’s resources $S$ (i.e., the total amount of available resources is $S + \xi_B$). The next proposition characterizes how the additional earmarked funds for exclusive use in the $B$-type service affect the optimal levels of advisory and service delivery efforts for an NPO that offers both types of services. In order to focus...
Figure 6. (Color online) Optimal Portfolio of Services with Earmarked Funds

![Graphs showing optimal portfolio of services for different values of \( \rho \)]

Note. Parameters are the same as in Figure 3 with \( \xi_B = 4 \).

on the effect of earmarked funds, we consider situations where the mix of clients is balanced (i.e., \( \gamma = 1/2 \)), while note that similar insights continue to hold for a general case (see details in the e-companion).

**Proposition 4.** (Effect of Earmarked Funds (\( \xi_B \)) on Optimal Efforts). Suppose \( \xi_B > 0 \) and \( \gamma = 1/2 \). Then,

i. \( e^*_A \) and \( e^*_B \) are increasing in \( \xi_B \).

ii. There exists a unique threshold \( \tilde{\xi}_B > 0 \), such that \( e^*_C \) is nondecreasing in \( \xi_B \) if \( \xi_B \leq \tilde{\xi}_B \), and nonincreasing in \( \xi_B \) otherwise.

When the level of additional earmarked funds is low (i.e., \( \xi_B < \tilde{\xi}_B \)), it is optimal for the NPO to allocate greater resources than the earmarked funds to the \( B \)-type service delivery effort. This implies that the NPO can follow its optimal strategy by allocating the remaining resources \( S \) across all efforts. However, when the level of additional earmarked funds is high (i.e., \( \xi_B \geq \tilde{\xi}_B \)), the NPO will not spend any of the remaining resources \( S \) on the \( B \)-type service delivery (i.e., constraint (8) binds), and will allocate \( S \) to only \( A \)-type service delivery effort and advisory effort. In that case, Proposition 4 shows that while optimal \( A \)-type effort increases in \( \xi_B \), the optimal advisory effort decreases in \( \xi_B \). In this case, by reducing advisory effort, the NPO can utilize more resources for the \( A \)-type service delivery, which enables the NPO to also hasten the progress of \( a \)-type clients. This approach minimizes the overall loss of impact due to delays and allows the NPO to provide faster service to all clients.

**Proposition 5.** (Optimal Portfolio of Services with Earmarked Funds). Suppose \( \gamma = 1/2 \) and \( 0 < \xi_B < \tilde{\xi}_B \) where \( \tilde{\xi}_B \) is as defined in Proposition 4. There exists a unique threshold \( \Phi_{\xi} \) such that it is optimal for the NPO to offer:

i. both types of service when \( \frac{1}{B} \geq \Phi_{\xi} \), and

ii. only \( B \)-type service when \( \frac{1}{B} < \Phi_{\xi} \).

Further, the thresholds \( \Phi_{\xi} \) is decreasing in \( \rho \) and in \( l \).

Proposition 5 characterizes the optimal portfolio of the NPO that receives additional earmarked funds for the \( B \)-type service. It shows that, even in the presence of earmarked funds, it is optimal for the NPO to offer both types of services when the relative impact \( (l/w) \) is high, the loss of impact from not serving a fraction of clients \( (\rho) \) is high, or the latency factor \( (l) \) is high. These results are consistent with our findings in Proposition 1. Note that although Proposition 5 formalizes these result for the case where \( \xi_B \) is below a threshold, extensive numerical analyses show that the effect persists for general \( \xi_B \).

Figure 6 illustrates the optimal service portfolio when the NPO receives additional earmarked funds \( (\xi_B) \) and the mix of clients may not be balanced (i.e., \( \gamma \geq 1/2 \)). As the amount of earmarked funds increases, the region in which only \( A \)-type service is optimal shrinks and eventually disappears. The figure also shows that the NPO could find it optimal to not accept earmarked funds (and offer only the \( A \)-type service) when the amount of such funds is relatively small, the fraction of \( a \)-type clients is sufficiently high, and the cost of delay is in an intermediate range. This may seem counter-intuitive at first, given that the earmarked funds are in addition to the available resources \( S \). The reason it is still optimal to offer only the \( A \)-type service (and not offer the \( B \)-type service) is that with the small amount of earmarked funds and high cost of delay, the delay experienced by \( b \)-type clients can lead to a significant loss of impact. This implies that, for many NPOs, accepting earmarked funds is optimal only when a substantial amount of earmarked funds is available to begin with, which is consistent with the observations from practice (Foundation Group 2018). Figure 6 also shows that as \( \rho \) increases, the region where offering both services is optimal enlarges, while the \( B \)-only region shrinks, which is consistent with our observations in Figure 3.

Next, we discuss the comparison between impact-focused and output-focused NPOs in the presence of earmarked funds. An output-focused NPO will always welcome earmarked funds and offer both types of
services, even in cases where an impact-focused NPO may not take such funds. Figure 7 compares the total expected impact of the impact- and output-focused NPOs with respect to earmarked funds (note that the initial flat part of TEI^I is due to the optimality of offering only Λ-type services). The social impact created by both types of NPOs increases as the amount of earmarked funds increases (since the NPO has more resources overall). However, the gap between them decreases as ξ_B increases. When the amount of earmarked funds is small, the impact-focused NPO can generate a higher social impact by prioritizing speed of service over the volume of clients that receive resolution by offering a single type of service. However, as ξ_B increases, the additional earmarked funds make it desirable for the impact-focused NPO to also prioritize the volume of clients by offering both services (as shown in Figure 6). As a result, when ξ_B is large, both types of NPOs can produce the same social impact.

Overall, our analysis in this section shows that NPOs should revisit their service and portfolio design when they receive additional earmarked funds for delivering a service. Specifically, when receiving a substantial amount of earmarked funds, it becomes optimal for NPOs to diversify their service offerings. In addition, they should invest less in advisory activities, but invest more in service delivery.

5. An Illustrative Case Study Based on the Example of Daya

In this section, we illustrate how our results can apply in practice by focusing on the specific context of domestic abuse and using the example of Daya. Because Daya serves clients who typically face different categories of abuse, including physical, financial, and emotional, we first explain how our model and results can be generalized to the case where the NPO can serve multiple client types in Section 5.1. We then estimate values for model parameters using available data and present our findings in Section 5.2.

5.1. Generalized Model of Service and Portfolio Design Corresponding to the Example of Daya

Here, we present the model for the case where the NPO serves m types of clients (with m ≥ 2) and can offer m ≤ n types of services. We denote the proportion of j-type clients by γ_j ∈ (0, 1), for j ∈ {1, 2, ..., n}, where \sum_{j=1}^{n} γ_j = 1. Consistent with the model in Section 3, when clients of each type reach the appropriate service delivery state (which is state j for j-type clients), they can transition to the resolution state R with probability \mu(e) in each period. However, following the advisory state, a client may transition to any one of the n – 1 mismatched states with probability (1 – \theta(c_e))/ (n – 1). Accordingly, we generalize the NPO’s optimization problem introduced in Section 3.2 to the case with multiple types of clients and present the closed-form characterization of the optimal efforts in the e-companion. We show that, as the number of service offerings (m) increases, it becomes optimal for the NPO to exert lower advisory effort and also lower service delivery efforts. The next proposition characterizes the optimal number of services that the NPO should offer in a simple case where the mix of clients is balanced and the latency factor is equal to one.

Proposition 6. (Optimal Number of Services). Suppose there are n ≥ 2 types of clients with γ_j = \frac{1}{n} and l = 1. Then,

i. there exists a unique m∗ \in [1, n], such that it is optimal for the NPO to offer m∗ types of services.

ii. m∗ is nonincreasing in w, and it is nondecreasing in S and in ρ.

Proposition 6 shows that there is a unique number of service offerings that maximizes the NPO’s total expected social impact. Combining this with the result that c_C declines as m increases, we obtain a nuanced picture of the NPO’s optimal strategy in maximizing its social impact: NPOs should prioritize investing in advisory activities over diversifying their service offerings when delays in achieving resolution can significantly blunt the social impact created. However, when more resources are available or the loss of impact factor is high, they should prioritize diversifying their services over investing in advisory activities. This complements the insights from Section 4. We next generalize these insights by focusing on the specific context of domestic abuse where the client mix is...
not balanced, and costs of delay vary among different client types.

5.2. Optimal Service and Portfolio Design in the Context of Domestic Abuse

In this section, we estimate model parameters using the example of Daya and data from academic sources and national agencies that track domestic abuse statistics. Daya’s clientele comprises clients who predominantly face one of the following five types of abuses: (i) Verbal or emotional abuse; (ii) Physical and sexual abuse; (iii) Financial abuse; (iv) Immigration-related abuse; (v) Abuse by in-laws or other family members of the spouse. Clients are first taken through a deep intake session where their issues are discussed and clients select one of the service plans offered by Daya. Each of these service plans is best suited for one type of abuse, and delivered until clients achieve their long-term resolution. Thus, for our numerical examples, we consider an NPO that can serve \( n = 5 \) types of survivors of domestic abuse. Let \( j \in \{1, 2, \ldots, 5\} \) denote each of the five types of clients listed above. Available data based on reports on domestic abuse allow us to estimate the parameter values as noted below Table 1 (see details in the e-companion).

Next, we solve the NPO’s generalized optimization problem introduced in Section 5.1 by using the estimated values of parameters to compute the optimal number of services and allocation of resources among service stages. These decisions depend on the financial resources available, which can vary among NPOs. We therefore study how our findings may change as a function of the NPO’s resources. Our findings are summarily provided in Table 1 for \( S \) ranging from $40,000 to over $55,000 per month, which is an appropriate range of financial resources for the type of NPOs we study (Snyder 2016).

When the NPO has a limited amount of resources \((S \leq 41,250)\), it is optimal for the NPO to offer only three out of the five possible services and also to not invest in advisory effort. At the other extreme, if \( S > 53,000 \), it is optimal to offer all five services and provide advisory support as well. This is because having more resources allows the NPO to serve a larger fraction of clients without compromising the speed with which it serves them. When resources are at intermediate levels \((41,250 < S \leq 53,000)\), we see that the NPO should actively manage the trade-off between serving more clients (by offering more services) and serving them fast.

As the amount of resources increases beyond $41,250 per month, it is optimal for the NPO to offer more advisory services to its clients rather than increasing the number of services. This allows the NPO to minimize the delay costs of the subset of clients it serves. Only after the amount of resources reaches a higher monthly level of $47,500 should the NPO expand its service portfolio (increasing \( m^* \) to 4). This allows the NPO to serve more clients without significantly increasing the incidence of mismatches. Finally, when the NPO’s resources increase above the threshold of $53,000, it is able to provide all five services along with advisory support for clients. All in all, by illustrating variation of \( m^* \) and \( c_G^* \) with respect to the resource level \( S \), our numerical analysis demonstrates how NPOs should balance efforts across various stages of service offering to maximize their net social impact.

As a summary measure of the NPO’s effort allocation decisions, we can also consider the optimal ratio of advisory to service delivery efforts; we define this as \( R_G = c_G^*/(S - c_G^*) \). We find that—with each optimal portfolio decision \( m^* \)—the ratio of advisory to service delivery efforts is strictly increasing in \( S \). However, due to the reallocation of resources when new services are added to the portfolio, the range of \( R_G^* \) varies across different service portfolios.

To illustrate the effect of the loss of impact factor \( (\rho) \), we calculate the total expected social impact for different values of \( \rho \). Similar to the finding shown in Figure 5(a), we find that the social impact generated by the NPO reduces in \( \rho \) for a given portfolio of services. In addition, we find that an increase in \( \rho \) makes it more desirable for the NPO to diversify its service offerings (even when it has a limited amount of resources). Table 2 presents the NPO’s optimal portfolio and the corresponding thresholds on the amount of resources for different values of \( \rho \). In particular, it becomes optimal for the NPO to offer all five services when its resources increase above $52,750 for \( \rho = 0.5 \) and above $52,000 for \( \rho = 2 \), which are lower than the corresponding threshold of $53,000 for \( \rho = 0 \). Moreover, when \( \rho = 2 \), there are no instances in which the NPO offers only three services.

We next discuss the comparison between impact-focused and output-focused NPOs. In the context of
this case study, an output-focused NPO would offer all five services irrespective of the amount of available resources. We find that an impact-focused NPO that has a comparable amount of resources as Daya (which is about $50,000 per month) can generate at least 14.7% higher social impact than the output-focused NPO of the same size. Although the impact-focused NPO will not serve a fraction of its clients, it can help those clients who receive services to reach resolution at least 18.6% faster than an output-focused NPO.

In practice, not all NPOs adopt impact-focused objectives because of the difficulty of tracking quantifiable measures of impact. For instance, NPOs may need to adopt client relationship management tools, such as Osnium and Penelope, which help them with collecting, monitoring, and reporting complex essential client data (Athena Software 2020, Osnium Inc. 2020). Our analysis demonstrates that the additional operational and societal benefits of maximizing the social impact instead of the output can justify upfront investments in such tools.

### 6. Extensions

#### 6.1. Clients Requiring Both Types of Services

In this section, we extend our model and analysis to situations where some clients require both types of services to achieve long-term resolution. We denote the fraction of clients who require both services by $\eta \in [0, 1]$. To simplify the exposition, we present formal results, technical details, and figures of this extension in the e-companion.

We find that when $\eta$ increases, clients experience longer delays and thus the impact produced by both impact- and output-focused NPOs reduces. When $\eta$ is small, the impact-focused NPO offers only a single type of service, while the output-focused NPO offers both services. Given that the proportion of clients who need both services is small, an impact-focused NPO can generate higher social impact by prioritizing the speed of resolution. However, when $\eta$ is large, the impact-focused NPO also offers both services and the two types of NPOs can perform the same. In addition, we study how the prevalence of clients needing both types of service interacts with the effect of earmarking funds. When the cost of delays ($\omega$) is low, the impact-focused NPO offers only the $A$-type service when $\eta$ is small and both types of services when $\eta$ is high. In this case, the impact gap between the two types of NPOs declines as the level of earmarked funding increases. The reason is that a higher $\xi_B$ is beneficial to the output-focused NPO but not to the impact-focused NPO when $\eta$ is small (as the latter offers only the $A$-type service). On the other hand, when $\omega$ is high, the impact gap between the two types of NPOs increases as the level of earmarked funds increases. In this case, the impact-focused NPO offers either the $B$-type service or both services. As a result, an increased availability of earmarked funds is liberating for the NPO, as it can utilize this additional funding to ensure $B$-type clients achieve resolution with smaller delays even when more clients need both types of service.

Overall, a higher fraction of clients who require both types of services makes the two types of NPOs to produce the same impact. However, whether the availability of earmarked funding attenuates or magnifies this effect depends on the optimal portfolio of the impact-focused NPO.

#### 6.2. Complexity of Clients’ Needs

In this section, we extend our model and analysis to situations where some clients have relatively more complex needs and therefore require further attention from the NPO during service delivery. We denote the fraction of clients who need further attention during service delivery by $\delta \in [0, 1]$. Accordingly, we introduce additional service delivery states $A_c$ and $B_c$, such that those $a$-type ($b$-type) clients who have complex needs proceed from their matched state $A$ ($B$) to state $A_c$ ($B_c$) before reaching resolution. To simplify the exposition, we present formal results, technical details, and figures of this extension in the e-companion.

We find that when more clients have complex needs, the reallocation of resources to help them achieve resolution comes at the cost of increased delays for all clients. This causes a decline in the social impact produced by the either type of NPO (impact- or output-focused). An impact-focused NPO, however, can mitigate this adverse effect by offering only a single type of service. Therefore, a gap emerges between the social impacts of the two types of NPOs above a threshold value of $\delta$. Further, this gap also increases steadily with $\delta$, reflecting the unmitigated adverse effect of $\delta$ on the output-focused NPO. The prevalence of complex needs among clients

### Table 2. Optimal Portfolio Design Based on the Numerical Case Study with $\rho \geq 0$

<table>
<thead>
<tr>
<th>Portfolio of services ($m'$)</th>
<th>$m'=3$</th>
<th>$m'=4$</th>
<th>$m'=5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss of impact factor ($\rho$)</td>
<td>Thresholds on monthly resources (in 1,000 USD)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho = 0$</td>
<td></td>
<td>40 &lt; $S$ ≤ 47.5</td>
<td>47.5 &lt; $S$ ≤ 53</td>
</tr>
<tr>
<td>$\rho = 0.5$</td>
<td></td>
<td>40 &lt; $S$ ≤ 43.75</td>
<td>43.75 &lt; $S$ ≤ 52.75</td>
</tr>
<tr>
<td>$\rho = 2$</td>
<td></td>
<td>–</td>
<td>40 &lt; $S$ ≤ 52</td>
</tr>
</tbody>
</table>

Note. Parameters are the same as in Table 1.
also interacts with the amount of earmarked funds, but the interaction depends on the context. When the cost of delays (\(w\)) is low, the impact gap between the two types of NPOs declines as the level of earmarked funding increases. In this situation, the availability of funds earmarked for offering the B-type service induces the impact-focused NPO to offer both services even at higher values of \(\delta\), and helps the output-focused NPO to improve its social impact even when \(\delta\) is high. In contrast, when the cost of delays (\(w\)) is high, the impact gap between the two types of NPOs increases as the level of earmarked funding increases. Here, the availability of the earmarked funds induces the impact-focused NPO to offer only the B-type service at even lower levels of \(\delta\), which leads to a larger impact gap between the two types of NPOs.

Taken together, we conclude that the greater prevalence of clients who have complex needs makes it even more important for impact-focused NPOs to optimize their service portfolio in a different manner from output-focused NPOs. However, the availability of earmarked funds may make this more or less critical depending on the optimal portfolio of the impact-focused NPO.

6.3. Clients Dropping Out Before Resolution

In this section, we extend our model and analysis to situations where some clients drop out before obtaining long-term resolution to their needs. That is, they lose the ability or willingness to fight their distress and stop seeking a solution for their problem. We denote the fraction of clients who drop out by \(r \in [0, 1]\) and capture such situations by introducing a dropout state \(D\). To simplify the exposition, we present formal results, technical details, and figures of this extension in the e-companion.

When a fraction of clients drop out before obtaining resolution, we continue to find that as the delay cost increases, it becomes optimal for the NPO to focus on delivery of only one type of service. Further, the threshold below which it is optimal to offer both types of services increases with an increase in the fraction of clients who drop out every period (i.e., as \(r\) gets larger). The reason is that, when \(r\) is large, the average number of clients in each service delivery state is lower. This frees up a greater amount of resources for advisory and service delivery efforts. As a result, the NPO can generate higher impact by spreading its resources between the two types of services even when the cost of delay is high.

We also consider the output-focused NPO’s optimization problem with dropout rate \(r \in [0, 1]\). Naturally, the presence of the dropout rate makes the output function of the NPO (i.e., number of clients who obtain long-term resolution) more sensitive to the efforts of the NPO. We find that it is generally optimal for the output-focused NPO to offer both services. In some cases, especially when the dropout rate \(r\) is high, it becomes optimal for the output-focused NPO to offer a single service which has a higher demand. When the dropout rate \(r\) is high, the NPO can maximize its output by offering a single type of service, as that allows the NPO to invest more in service delivery effort of the offered service, which in turn decreases the rate at which clients drop out from their best-suited service (i.e., \(r (1 - \mu (e_A))\)). When comparing the performance of the impact- and output-focused NPOs, we find that, when it is optimal for the impact-focused NPO to offer both services, it is almost always optimal for the output-focused NPO to also offer both services. Further, comparing the optimal advisory efforts of the two types of NPOs, we find that the optimal advisory effort of an impact-focused NPO is almost always greater than or equal to the optimal advisory effort of an output-focused NPO (i.e., \(c_G^A \leq c_G^O\)). Overall, these insights support the key results pertaining to comparison of impact- and output-focused NPOs (in Section 4.1).

6.4. Other Extensions

In this section, we extend our model and analysis to three additional situations, namely asymmetry in costs of delay, asymmetry in service delivery transition rates, and time-dependent transition rates. To simplify the exposition, we present formal results, technical details, and figures of this extension in the e-companion. We next briefly explain the key insights from these extensions.

We study the effect of asymmetry in costs of delay (i.e., \(w_A \neq w_B\)) on the NPO’s optimal service and portfolio design. Considering situations where \(w_A > w_B\), we find that NPO should either offer both services (when \(w_A\) is low) or focus only on delivering the B-type service (when \(w_A\) is large). If the NPO continues to offer both services as \(w_A\) increases, it should also exert a higher A-type service delivery effort. This results in fewer resources being available for advisory and B-type service delivery efforts, which adversely affects all clients. Therefore, when \(w_A\) exceeds a threshold, the NPO can generate a higher social impact by providing only the B-type service.

We also study the case where transition rates are different across the two services (i.e., \(\mu_A \neq \mu_B\)). We find the region in which it is optimal to offer only the A-type service is larger when the transition rate of the A-type service is high. In general, as the cost of delay increases, it is optimal to offer only the less-demanded B-type service. However, when \(\mu_A > \mu_B\), it is also optimal for the NPO to offer only the A-type service when the proportion of a-type clients is close to 0.5 and the cost of delay is large. This is an effect that can be explained only by the asymmetry between transition rates. Here, because of the lower transition rate of the B-type service, the NPO is unable to take advantage of faster resolution for a smaller set of clients as
Proposition 1 suggests. It is therefore optimal for the NPO to offer only the $A$-type service.

Additionally, we study situations where service delivery transition rates are time-dependent. Specifically, we define $k \in (0,1)$ as the rate of improvement in the clients’ transition probability, such that the probability that a client transitions from the service delivery state $j$ to the resolution state is $\mu_j(e^t) = 1 - k^t (1 - \mu e^t)$ for $j \in \{A,B\}$, where $t$ is the number of rounds the client has spent in that service delivery state. In this case, we continue to find that as the delay cost increases, it becomes optimal for the NPO to focus on delivery of only one type of service. Further, the threshold below which it is optimal to offer both types of services increases with an increase in the rate of improvement (i.e., as $k$ gets smaller). The reason is that, when $k$ is small, the average number of clients in each service delivery state is lower due to improved rates of clients transitioning to resolution state. As a result, the NPO can generate higher impact by spreading its resources between the two types of services even when the cost of delay is high.

7. Discussion and Conclusion

This paper studies the optimal service and portfolio design of nonprofit organizations (NPOs) that serve distressed individuals. Based on our experience and involvement with several NPOs, we realize these organizations operate under a complex combination of challenges such as limited funding, heterogeneity in clients’ needs, delays in achieving resolution for clients, and mismatches between clients’ needs and services provided. As such, in choosing the optimal portfolio of services to offer, these NPOs face a trade-off between the speed of resolution and the quantity of clients served. Further, with regards to the optimal service design, they face a trade-off between improving matching for incoming clients (via advising) and accelerating resolution for the matched clients (via service delivery). In this paper, we develop a model-based framework that can help nonprofits design portfolio and service processes for maximum social impact.

Our analysis generates the following first-order managerial insights for impact-focused NPOs that serve clients in distress: First, our results suggest that NPOs should be cautious about expanding their service offerings when funds are limited or the social costs of delayed resolution is high; on the contrary, diversifying the service portfolio is suitable when the social impact from distress resolution is high, or when clients who are not served have few outside options, or when substantial earmarked funds are available. Second, when more funds become available, we show that NPOs should first give priority to improving their intake processes and providing better guidance for clients so that they can choose the appropriate services, and only then consider expanding their service portfolio. This approach allows them to improve the operational efficiency of their existing service processes and help clients receive resolution to their needs promptly. Summarily, we identify when—and show how—NPOs can produce more social impact by optimizing their service portfolio.

We also compare the performance of impact- and output-focused NPOs. We show that output-focused NPOs that focus on serving the most number of clients offer more number of services and do not prioritize advisory effort, which could result in lower social impact. When the two types of NPOs perform differently, the additional impact produced by impact-focused NPOs is larger when the mix of client types is uneven, earmarked funds are minimal, only a few clients need both types of services, or when a large fraction of clients have more complex needs. Finally, we illustrate how the insights from this paper relate to practice through a numerical case study based on the context of domestic abuse. We show that by adopting impact-focused objectives, NPOs can generate notably higher social impact and help their clients reach their long-term resolution considerably faster than output-focused NPOs of the same size.

Our conversations with disaster relief and recovery organizations such as Team Rubicon and Project Para-link suggest that our model can also be applied to such organizations (Ludema and Johnson 2020), albeit with a modified emphasis on various elements. First, disasters demand urgent intervention and not serving clients may lead to loss of life, suggesting that the loss of impact from not serving a fraction of clients (i.e., our model parameter $\rho$) is likely quite high. Relatedly, disaster-relief organizations aim to serve as many beneficiaries that they can reach; this suggests that output-focused measures could be appropriate (Ebrahim and Rangan 2014). In fact, our results support this strategy. It is also important to note that disasters and distress are not independent problems in society. For instance, early observations from the COVID-19 pandemic shows that domestic violence has risen due to the forced lock-downs in societies all over the world (Bettinger-Lopez and Bro 2020). As disaster-relief organizations focus on immediate problems such as distributing protective equipment and ventilators, organizations like Daya and Georgia Works will inevitably face greater challenges in the longer term as well. We hope future researchers in Operations Management will pursue these research opportunities.

Adopting appropriate measures of impact could also help NPOs influence donors. Donors have increasingly emphasized the measurement of impact as they “want to know whether their funds are making a difference or might be better spent elsewhere” (Ebrahim and Rangan 2014). In order to ensure that donors understand and remain supportive of their execution, it is important for NPOs to maintain a high level of
transparency about their goals and plans with donors (Privett and Erhun 2011). Our study also yields two insights for benefactors: (i) if they want to influence an impact-focused NPO’s actions through earmarking, they should donate a large amount (perhaps by making fewer consolidated donations rather than spreading their funds across many NPOs); (ii) if the benefactor is allocating a small amount of funds, they should simply make a nonearmarked donation that would allow the NPO to maximize impact in a manner of their choosing. A promising future research direction would be to study the design of audit mechanisms to alleviate any adverse effect of misalignment between the donor’s and NPO’s objectives. Also, studying situations where different NPOs have to compete for funds from donors, and examining the implications of that competition on NPOs’ service and portfolio designs can be informative.

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Endnotes

1 The examples in the paper are based on direct interactions between the authors and the NPOs.

2 Georgia Works conveys this focus directly to prospective clients in all forms of communication, including their website http://www.georgiaworks.net/about-1.

3 In Section 6.3, we extend our model and analysis to situations where some clients drop out before obtaining resolution to their needs. In such situations, there is a unique combination of optimal efforts that maximizes the NPO’s output. We show that the overall insights from the comparison of impact- and output-focused NPOs remain robust in such a model.

4 As shown in Lemma EC-6 in the e-companion, when $\xi_s$ is above that threshold, we can only obtain implicit solutions for optimal efforts, which limits the analytical characterization of the optimal portfolio.

5 Note that parameter estimations can depend on geographical and social factors. For instance, NPOs such as Sakhi (New York City) and Raksha (Atlanta) share Daya’s mission but operate in very different locations.

References


