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Economics of Free Under Perpetual Licensing: Implications for the Software Industry

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In this paper, we explore the economics of *free* under perpetual licensing. In particular, we focus on two emerging software business models that involve a free component: *feature-limited freemium* (*FLF*) and *uniform seeding* (*S*). Under *FLF*, the firm offers the basic software version for free, while charging for premium features. Under *S*, the firm gives away for free the full product to a *percentage* of the addressable market *uniformly* across consumer types. We benchmark their performance against a conventional business model under which software is sold as a bundle (labeled as "charge for everything" or *CE*) without free offers. In the context of consumer bounded rationality and information asymmetry, we develop a unified two-period consumer valuation learning framework that accounts for both word-of-mouth (WOM) effects and experience-based learning, and use it to compare and contrast the three business models. Under both constant and dynamic pricing, for moderate strength of WOM signals, we derive the equilibria for each model and identify optimality regions. In particular, *S* is optimal when consumers significantly underestimate the value of functionality and cross-module synergies are weak. When either cross-module synergies are stronger or initial priors are higher, the firm decides between *CE* and *FLF*. Furthermore, we identify nontrivial switching dynamics from one optimality region to another depending on the initial consumer beliefs about the value of the embedded functionality. For example, there are regions where, ceteris paribus, *FLF* is optimal when the prior on *premium* functionality is either relatively low or high, but not in between. We also demonstrate the robustness of our findings with respect to various parameterizations of cross-module synergies, strength of WOM effects, and number of periods. We find that stronger WOM effects or more periods lead to an expansion of the seeding optimality region in parallel with a decrease in the seeding ratio. Moreover, under *CE* and dynamic pricing, second period price may be decreasing in the initial consumer valuation beliefs when WOM effects are strong and the prior is relatively low. However, this is not the case under weak WOM effects. We also discuss regions where price skimming and penetration pricing are optimal. Our results provide key managerial insights that are useful to firms in their business model search and implementation.

Keywords: software; freemium business models; versioning; seeding strategies; product sampling
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1. Introduction

Software markets experienced a dramatic change over the past couple of decades. On one hand, information technology performance-to-price ratio increased significantly and user interfaces became much more friendly, accelerating the adoption of computers toward supporting activities on both professional and personal levels, which, in turn, led to software becoming ubiquitous to the functioning of our society. According to Datamonitor (2011), the size of the rapidly growing global software market was $265.4 billion in 2010 and is estimated to reach $356.7 billion by 2015. On the other hand, Internet penetration and usage grew at a staggering rate, with current statistics reporting over 2.4 billion Internet users and over 996 million interconnected Internet hosts worldwide (Internet World Stats 2013, Internet Systems Consortium 2013). This led to the emergence of online software distribution models (e.g., online software marketplaces such as Apple App Store, Google Apps Marketplace, or repositories such as SourceForge.net), online software consumption models (e.g., software-as-a-service offerings such as Salesforce.com CRM suite or GE's Centricity Electronic Medical Records service), as well as online feedback models that facilitate the propagation of word-of-mouth (WOM).

In this rapidly evolving software industry, in parallel with conventional business models, whereby the entire product or each separate module comes with an associated price (e.g., various versions of Microsoft Windows operating system), a new software business model, called *freemium* (Anderson 2009), has emerged. This model combines "free" and "premium" consumption in association with a product. In a nutshell, the model involves giving away for free a certain level or type of consumption while making money on premium consumption. Freemium models are spreading quickly in the software industry, especially among

Web start-ups (Miller 2009). Freemium models are seen in practice predominantly under two flavors: *feature-limited freemiums* (*FLF*) and *time-limited freemiums* (*TLF*). The difference between these two models is on the scope and licensing terms of the free offering. *FLF* models involve offering a basic version of the product with limited functionality for free, while charging for additional features in the premium version. Under *FLF*, the free version comes under *perpetual* licensing, meaning that the users can use indefinitely that limited functionality. This practice is very common for mobile apps and also existed for many years in the video game industry. On the other hand, *TLF* gives access to the complete functionality but only for a limited period of time. For example, Adobe Photoshop CS 6 and Microsoft Office Professional Plus 2013 come with a 30-day and a 60-day free trial, respectively. The feasibility of the freemium models is predicated on several features of software products. First, software products nowadays are increasingly built using a modular architecture which, in turn, facilitates grouping together, separating, or locking certain features. Second, software products are digital goods with negligible marginal reproduction cost, that can be provided in unlimited supply and can be "shipped" via relatively cheap online distribution channels. This way, the great majority of interested customers have access to the free consumption opportunities embedded in the freemium offering. Third, software products often belong to the category of experience goods. By trying (sampling) the product or part of it before committing to any purchase, consumers could learn more about the quality and other attributes (such as performance, functionality, interface, and features) of the software, capabilities of related modules, compatibility issues, hardware requirements, etc.

Although both *FLF* and *TLF* are seen in practice, there exist markets where *TLF* is not supported. The most prominent of them is Apple App Store, which, as of the end of October 2013, served around 700 million sold iOS mobile devices globally, exhibiting over 1 million available apps and over 60 billion app downloads (Claburn 2013, Ingraham 2013). As of June 2013, 93% of the iOS apps were downloaded every month (Whittaker 2013). Apple's entry into the mobile telecom space can be considered one of the most disruptive in tech industry, completely reshaping this particular sector. Apple's success in the mobile industry hinges heavily on harnessing the platform potential of mobile devices, vastly increasing their versatility beyond simple communication tasks. This has been accomplished by supporting an ecosystem of developers who could provide value-adding apps on top of the iOS platform. Although Apple supports revenue sharing (with over $13 billion paid to app developers to date; Ingraham 2013), it retains full control over which applications are allowed in the App Store and the ways in which they can be provided. Moreover, App Store is the only authorized distribution channel for apps for iOS devices. In this market, Apple restricts the ability of developers to demo their applications. The only two ways this can be done are either via a *lite* version with limited functionality and a separate full-functionality version, or by offering the application for free and charging for *in-app purchases*. In both cases, once users gain access to the basic version for free, that access is granted in perpetuity; however, they do have to pay an additional fee to access premium functionality or content. As of September 2013, in the United States, 86% of the revenue in the Apple App Store was generated by in-app purchases inside free apps (Distimo and MEF 2013). In general, time limited trials where the product locks after the trial expiration are not supported in this marketplace. A recent change in iTunes terms stipulates that publishers can offer free time-limited trials for certain paid in-app subscriptions. Nevertheless, these terms apply only to apps that give access to subscription-based content (Crookes 2012). On the other hand, developers can initiate short promotional campaigns through which they can offer their app for free and under perpetual licensing, but the offer is no longer available once the campaign ends. As such, these short-term markdowns are similar to *random seeding* practices as only a few customers who happen to come across the information during that time window of the campaign can take advantage of it.

As illustrated in the above example, it is of managerial importance to some software developers (e.g., developers of mobile apps) to understand the value of *free* offers when *both* free and non-free offers come under *perpetual* licensing. In this paper, we set out to explore this research question.[1] In the context of our problem, we focus on two relevant software business models that involve a free component: *FLF* and uniform seeding (*S*). We benchmark the performance of these two models against a conventional business model whereby the developer sells the software as a bundle without any free offer, which we label as "charge for everything" (or *CE* in short). We seek to characterize regions where each strategy is optimal and explore how various market conditions affect these regions and the associated business models.

Our study brings forward several contributions. First, on the modeling side, in the context of modular software (additional premium content can also be considered as an add-on module), we propose a two-period consumer valuation learning framework that

[1] Although *TLF* is not considered in this analysis, it represents a widely used model in some software markets and we direct interested readers to Cheng and Tang (2010) and Dey et al. (2013) for recent studies involving this model.

accounts for both WOM effects and experience-based learning induced by cross-module synergies. Customers do not know in the beginning their true valuation for the functionality embedded in each module. They may be off in their initial estimate by different factors relative to basic versus premium functionality. The learning framework is considered in the context of consumer bounded rationality induced by the lack of information in the market and the cognitive complexity that would underlie keeping track of all potential ways in which the market might unfold in the future. Our setup is general, allowing for the consideration of both imperfect and perfect learning due to WOM. Using this unifying framework, we can model all three strategies (*CE*, *FLF*, *S*), capturing the similarities and differences among them. This, in turn, allows us to compare and contrast market outcomes induced by these strategies.

On the theory side, this paper pushes forward the research agenda on economics of free on several dimensions. This study goes beyond comparing *FLF* and *S* to *CE*. To the best of our knowledge, it is the first study to contrast *FLF* and *S*. When firms can choose among several ways to take advantage of offering free consumption, it is important to understand *which* of the available options yields the highest profit. Under constant pricing and moderate strength of WOM signals, we derive the equilibrium for each model (*CE*, *FLF*, *S*) and fully characterize conditions under which each model is best. Furthermore, we identify nontrivial switching dynamics from one optimality region to another depending in complex ways on the initial consumer beliefs about the value of the embedded functionality. For example, in regions where customers do not initially place a high value on the bundle, increasing consumer priors on the value of *premium* functionality may induce the firm to switch from *S* to *FLF* to *CE* and, ultimately, back to *FLF*. Thus, there are regions of the parameter space where, all other things equal, *FLF* is optimal when the prior on premium functionality is either relatively low or high, but not in between. Such switches can also be accompanied by transitions back and forth between capitalizing and not capitalizing on WOM effects. These observed switching patterns happen within reasonable ranges of cross-module synergies. On the other hand, if customers believe from the start that there is enough value in the bundle, then increases in the initial valuation of the premium functionality will not induce the firm to change strategy. We also explore the sensitivity of our results with respect to various parameterizations of cross-module synergies, strength of WOM effects, number of periods, and dynamic pricing. Although all business models can capitalize on WOM effects, we find that under stronger WOM effects or more periods, the parameter space under which seeding is optimal increases in parallel with a decrease in the optimal seeding ratio. Moreover, under dynamic pricing, for *CE* (and *FLF*), we uncover a nontrivial price behavior in the second period—it may be decreasing in initial consumer valuation beliefs when WOM effects are strong, but this is not the case under weak WOM effects. In addition, comparing the perpetual license prices in different periods, we discuss conditions under which the developer should engage in either price skimming or penetration pricing strategies. For completeness of the analysis, we also explore the special case of perfect learning via WOM. Tangentially, since *FLF* can be seen as a particular case of versioning, our study also uncovers novel conditions when versioning can be optimal in the absence of marginal costs.

In addition, our paper makes an important step toward solving a considerably more difficult problem. Namely, developers have to initially decide on a complex software production and marketing strategy that *jointly* involves development (how much functionality to build and how to split it between modules), advertising (aimed at helping customers form priors in the market), and which business model to follow in the market (*CE*, *S*, *FLF*). We are not aware of any study that comprehensively addresses this research question in the context of multiperiod product life and WOM effects. If we conceptualize a solution to this problem based on backward induction, the current paper presents a solution to the last stage—taking as given the functionality in each module and the consumer priors induced by the prerelease advertising campaigns, we solve for the optimal business model (among the three models considered). Often times, firms reevaluate their strategies along the way. As such, this paper solves developers' problem close to market release, once all other efforts are spent and price and commercialization models remain the major levers to shape demand. The insights derived are of significant managerial importance as they help firms determine whether or not to undertake certain investments in development and advertising[2]—knowing what revenue to expect under each outcome allows developers to consider what would be the optimal use of their resources. In particular, as mentioned above, in certain regions of the parameter space, advertising campaigns that boost customer prior valuation for premium functionality need to be paired with nontrivial decisions to switch from one business model to another. If they are aware of the need to switch the business model as well in addition to changing the price in response to outcomes

[2] Although advertising has been a visible tool in traditional software markets, we also start seeing it factor more in the realm of mobile apps. A recent survey study identified that mobile app developers who were successful in generating over \$50 K in revenues from their apps had allocated on average marketing budgets of nearly \$30 K per product (App Promo 2012).

of ad campaigns, firms can be more successful in commercializing their products.

The rest of the paper is organized as follows. Section 2 presents a summary of the relevant literature. In §3, we introduce the general modeling framework. In §4, we derive the optimal strategy of the software firm under constant pricing and explore its sensitivity with respect to various parameters. In §5 we extend our analysis to dynamic pricing. In §6, we summarize the conclusions of this study and present directions for further research. For brevity, all proofs and additional discussions have been put in the online supplement (available as supplemental material at http://dx.doi.org/10.1287/isre.2013.0508).

2. Literature Review

In this study, we draw mainly on three streams of research in IS economics and marketing: (i) product sampling and free demonstration, (ii) software versioning, and (iii) market seeding.

The first stream of literature relevant to our work is related to product sampling and free demonstration. These well-studied marketing strategies are particularly appealing to digital goods, many of which are *experience* goods whose value is learned by customers via trying the good itself or a version of it (Chellappa and Shivendu 2005). Under *FLF*, customers adjust their priors on the value of premium features based on experiencing basic functionality. Therefore, in digital goods markets, net of advertising, WOM effects, or direct network effects, firms can still influence individual consumers' product value expectations via free offers. Acknowledging this effect, the literature on product sampling and free demonstration explores how firms can influence adoption by educating (some of) the customers on the value of the product. One line of work (e.g., Jain et al. 1995, Heiman and Muller 1996, Heiman et al. 2001) accounts for network effects and establishes that customers change their priors on the product value after sampling it. However, most of these models consider sampling and free demonstrations made available only to a limited audience because of substantial replication and distribution costs associated with physical goods. Bawa and Shoemaker (2004) consider the extreme case where samples are offered to the entire addressable market. In all of the above models, price is treated as exogenous. Chellappa and Shivendu (2005), Cheng and Liu (2012), and Dey et al. (2013) extend this line of work by exploring sampling under endogenous pricing. The former study models a vertically differentiated market for digital goods where consumers do not know the true product value but can sample the goods through pirated versions. The latter two studies present an analysis of the optimality of *TLF* models where the free component is not offered under perpetual licensing. In these models, customers learn gradually the value of the product throughout the free trial and the length of the free trial is endogenized in the problem. Beyond a certain point, the free trial is discontinued. Apart from differences in licensing schemes (in our paper we focus on markets where the free offer comes under perpetual licensing), our framework is different on another dimension as it incorporates WOM effects that influence the valuation learning. Cheng and Liu (2012) consider network effects at utility level whereas Dey et al. (2013) abstract away from either WOM effects or utility-level network effects. In our model, the magnitude of belief updating based on WOM effects depends not only on WOM strength factor and network size, but also on how far the initial priors were from the true value for each customer.

The second stream of extant work relevant to our research is on software versioning, which in turn draws on the well-established economics literature on second-degree price discrimination (e.g., Mussa and Rosen 1978). In *FLF* models, vertical differentiation is achieved by offering the lower "quality" version (where quality can be measured in terms of performance, functionality, or content limitations) at no charge. There exists a rich literature (e.g., Raghunathan 2000; Bhargava and Choudhary 2001, 2008; Wei and Nault 2005; Chen and Seshadri 2007; Jones and Mendelson 2011; Shivendu and Zhang 2011, 2012; August et al. 2013; Chellappa and Jia 2013; Chellappa and Mehra 2013) that studies optimal software versioning under various utility structures and market assumptions. Ghose and Sundararajan (2005) complement this line of work by empirically estimating the extent of quality degradation associated with software versioning, using a quadratic utility structure. Wu and Chen (2008) further show how versioning can be used to deter digital piracy. Riggins (2003) and Cheng and Tang (2010) investigate cases where the low-quality product is offered for free in the context of unique Web content and software products, respectively. In the former study, the firm monetizes giveaways through advertising revenues generated by the users of free websites, whereas in the latter (extending the models in Conner 1995 and also adding an aggregate consumer usage cost) the firm trades off consumer valuation upshift due to positive network effects versus lost sales due to giveaways of the low-quality version. In Cheng and Tang (2010), a critical condition for the optimality of *FLF* strategy is the presence of strictly positive software usage costs at the individual consumer level. We complement this line of work by showing that *FLF* can also be optimal under negligible usage cost when consumers undergo WOM-based and experience-based valuation learning.

A common assumption of all the above reviewed versioning models is that customers have full information regarding the value of each product version, and they self-select into purchase groups according to the menus of price and quality offered by the vendors. Wei and Nault (2013) extend this line of work to scenarios of imperfect information on the consumer side and consider experience-based learning induced by adoption in a two-stage game where low-quality version adopters might decide in the second stage to upgrade to the high-quality version. In a similar vein, Dey and Lahiri (2013) explore how customers that find a basic version of an information good sufficiently attractive might proceed with the purchase of additional premium downloadable content. In both studies, customers learn by themselves and are not influenced by others in the valuation updating process. By employing a two-period framework with WOM effects, experience-based learning, and cross-module synergies to inspect how valuation learning and imperfect information on the consumer side affect the *FLF* offering, we bring a significant contribution to the existing literature on software versioning.

The third stream of extant work relevant to our research is on market seeding, whereby a ratio of the potential customers receive the full product for free. In that sense, seeding is another marketing strategy for the firm to influence consumer priors in order to jump-start adoption. Libai et al. (2005) explore how seeding affects marketing decisions when the product is introduced in multiple markets. Lehmann and Esteban-Bravo (2006) inspect optimal seeding under endogenized dynamic pricing, variable costs, and network externalities. Galeotti and Goyal (2009) analyze optimal seeding in a social network with a graph-like topology where decisions of individual customers are influenced by their immediate contacts. Jiang and Sarkar (2010) explore the effect of limited product giveaways on future adoption and net present value of future sales under an exogenous pricing rule. Dou et al. (2013) explore how seeding and social media features can be used in tandem to engineer optimal network effects. Libai et al. (2013) measure the value of seeding programs by separating the contribution of WOM effects between expansion and acceleration of the market. We extend these studies by exploring how seeding models fare compared to *FLF* models.

In summary, we contribute to the previous literature by integrating several of the above reviewed isolated streams of research. We develop a general multiperiod adoption framework with bidimensional learning based on both own experience (due to cross-module synergies) and WOM-based learning, capturing how consumer behavior evolves over time under different business models. Based on this unified framework, we formulate, solve, and analyze under perpetual licensing how freemium and seeding models fare against each other and against conventional for-fee models. In addition to the above mentioned contributions to each of the related streams of research, to the best of our knowledge, this is the first extensive analytical benchmarking of seeding versus freemium models, thus advancing our understanding of how firms can benefit from offering one form or another of *free consumption*. As discussed in §1, we uncover a host of insightful results that capture how the firm should alter its strategy depending on initial consumer beliefs, strength of WOM effects, and number of periods.

3. Preliminaries

3.1. Models

We assume that a firm has developed a software product that has two modules, A and B, and is exploring the most effective way to commercialize it. All software development costs are sunk. Basic functionality is coded in *stand-alone* module A, whereas module B represents an *add-on* that incorporates premium features or content that can only be accessed if module A is installed as well. The product (both modules) has a life span of two periods (after which it becomes obsolete or irrelevant) and it is offered under a *perpetual* license whereby, once customers acquire the license, they can use the software until the obsolescence horizon without incurring any additional charges in the future.

There are many business models under which the firm can commercialize the two contingent software modules. An exhaustive individual and comparative analysis of all these models is beyond the scope of this paper. This study belongs to the growing literature studying the economics of free offers. We focus on the evaluation of several strategies that involve some form of free giveaway, benchmarking their performance against that of strategies without free offers. The firm considers among the following models:

(a) *Charge-for-everything* (*CE*). The firm sells both modules *bundled together* (as one indivisible product). No consumer gets any functionality or content for free.

(b) *Uniform seeding* (*S*). The firm gives away for free the product (both modules) to a *percentage* k of the addressable market *uniformly* across consumer types at the beginning of period 1. The remaining potential customers, if interested, can only purchase both modules *bundled together* (as one indivisible product).

(c) *Feature-limited freemium* (*FLF*). The firm gives away module A as *freeware* from the beginning and makes money *only* on module B.[3]

[3] It is worth mentioning that, in practice, in the case of *FLF* models, sometimes premium functionality is bundled together with basic functionality in the premium version of the product creating a complete stand-alone solution (e.g., Adobe Reader versus Adobe Acrobat). However, if A is offered for free, whether the premium functionality is delivered as an add-on (only B) or as a separate stand-alone solution (both A and B integrated in one product), consumer choice is the same because of incentive compatibility constraints.

Under *FLF* and *S*, the free offerings are made at the beginning of period 1 and are provisioned under *perpetual licensing* terms. For all three business models, if a customer did not receive a module for free, she has the opportunity to purchase it in either period 1 or period 2.

3.2. Consumer Valuation Learning

We assume a normalized mass $m = 1$ of consumers with types θ distributed uniformly in the interval $[0, 1]$. A consumer of type θ derives *per-period* benefits $a\theta$ and $b\theta$ from using modules A and B, with $a, b > 0$. In that sense, type θ captures heterogeneity in the consumer willingness to pay (WTP) for quality and functionality per unit of time. The resulting linear utility model is similar to the one in Chen and Seshadri (2007), with the only difference that earlier adopters get to use the product more compared to late adopters. We consider a general setting where parameters a and b capture an aggregate benefit from the modules and are not necessarily linear in the number of features included in each module, as users might value various features differently. Without any loss of generality, we normalize $a = 1$. Moreover, we include in parameter b all additional *cross-module* benefits that arise from using functionality in A and B jointly, which otherwise are not available to customers using module A as a stand-alone (e.g., under *FLF* model). For simplicity, we assume no time discounting.

We assume that prior to product introduction (before the beginning of period 1) potential customers do not know the true value of parameters a and b, but they do have initial beliefs $a_0 = \alpha_a a = \alpha_a$ and $b_0 = \alpha_b b$ that govern their initial WTP (α_a, $\alpha_b > 0$). We consider the general case where α_a and α_b may be distinct from each other. These priors are formed based on prerelease promotional activities initiated by the firm and, in this paper, we consider these efforts spent and their associated costs sunk. For very novel features, customers might start with priors far from the true valuation due to the initial lack of information on usability/applicability of such features. In other cases, customers might understand the true valuation of functionality if it is built in a well-implemented product; however, if they use such valuation benchmarks but the coding turns out to be poor (which is hard to assess a priori) then the true value of the product in question might be overestimated at the very beginning (due to unanticipated future costs related to bugs and poor interfaces). On the other hand, if the product is just a slight variant of another product for which information already exists in the market (e.g., Angry Birds Rio is a variant of the popular Angry Birds game for mobile platforms, with a theme loosely inspired by the animation movie Rio produced by Blue Sky Studios), then consumers start with priors close to the true valuation for the product.

Over time, customers update their WTP for each module according to two distinct learning processes: (i) learning via *own experience*, and (ii) learning from *others* via WOM effects. We discuss below each of these.

3.2.1. Learning via Own Experience. Since many software products are experience goods, we assume that customers update their beliefs on the value of the product once they get to use all or part of the functionality. Such belief updating occurs independent of the opinion of other users and it is based solely on the consumer's own experience. We consider *direct* and *indirect* learning from own experience. Under the former learning process, we assume that customers immediately learn via use (i.e., directly) the true value (to themselves) of any functionality (module) that they have access to under the license (whether they purchased that license or received it for free). Thus, under *FLF*, all customers immediately update a_0 to $a_1 = a = 1$ at the beginning of period 1. Similarly, under *S*, at the beginning of period 1, seeded customers change their WTP by updating a_0 and b_0 to $a_1 = 1$ and $b_1 = b$, whereas unseeded customers enter period 1 with unchanged beliefs $a_1 = a_0$ and $b_1 = b_0$. Under *CE*, at the beginning of period 1, prior to any customer making any purchasing decision, there is no belief updating. Although consumers who obtained a perpetual license for a specific module in period 1 will not engage again in purchasing the same module in period 2, their own learning of the per-period value of the product gives them an opportunity to contribute to WOM effects in period 2, as will be discussed in §3.2.2.

In the *FLF* case, we also consider the possibility of *indirect* learning based on own experience with part of the software. More precisely, after trying module A, customers may adjust their WTP for premium functionality in module B contingent on the cross-module synergies between A and B but *without* having been exposed to module B (i.e., without having tried the premium functionality). We formalize this belief updating process as $b_1 = b_0 + \Delta(b)$. The magnitude and direction of $\Delta(b)$ can depend on many factors. For example, consider again the case of the infamous mobile game Angry Birds, developed by Rovio. This game is offered under a freemium model where users can play for free several levels and then have to pay to gain access to the remaining levels. However, there is a great similarity between levels in terms of interface and theme. Seeing a few levels gives the users a very good understanding of the value they will derive from the rest of the game. In that sense, b_1 would be close to true value b. At the other end of the spectrum, consider the example of the basic Matlab scientific programming software (not free) and the Optimization Toolbox add-on. One could argue that it is quite hard to understand the full benefit of functionality made available in the add-on based on the basic features of

Matlab. A case in between would be that of Adobe Acrobat. Adobe Reader, the free basic version, although without major editing capabilities, allows users to open PDF files created using the full Acrobat product, thus offering incomplete information about the value of premium functionality. Back to the general case, such indirect learning can shift prior beliefs for the value of module B in either direction as it depends on a host of factors, as will be later discussed at the end of §3.2.2. For simplicity, we assume $\Delta(b) = \delta_b b$. In the first part of the paper we assume δ_b is an exogenous constant. We later relax this assumption in §4.5 and show that insights continue to hold even when δ_b is correlated with some of the other parameters.

Relative to the time frame set in this paper (two discrete time periods), it is reasonable to consider the simplifying assumption that learning by own experience occurs in negligible time (compared to the length of one period that captures half of the useful life of the product). Nevertheless, we acknowledge that on a more granular time scale experience-based learning can be modeled as time dependent. For example, in a continuous time setting, the effect of time on experience-based learning has been considered in the study of the optimal length of free trials (e.g., Cheng and Liu 2012, Dey et al. 2013). Such an analysis is beyond the scope of this paper.

3.2.2. Learning via WOM. At the beginning of period 2, consumers who have not purchased the software in period 1 further adjust their beliefs about the value of the non-free modules based on WOM effects generated by the existing users of those *same* modules. WOM effects are commonly considered to affect adoption by influencing consumers' perceptions of product value and attributes (Mahajan et al. 1984, Ellison and Fudenberg 1995) and are increasingly disseminated over the Internet (Dellarocas 2003, Duan et al. 2008, Trusov et al. 2009). In the context of models *CE* and *S*, whereby the two modules are offered as a bundle, let N_t denote the cumulative number of consumers who *purchased* the full product by the end of period $t \in \{1, 2\}$ (excluding seeded customers). For model *FLF*, since module A is given away for free, customers can only purchase module B, which we indicate via subscript notation for the installed base associated with the premium module ($N_{1,B}$, $N_{2,B}$). Consumers who do not own a license (through purchase, giveaway, or seeding) before the end of period 1, will update their valuation perception for each module at the beginning of period 2 as follows:

$$CE:\ a_2 = a_1(1 - N_1^{1/w}) + aN_1^{1/w},\quad b_2 = b_1(1 - N_1^{1/w}) + bN_1^{1/w}, \tag{1}$$

$$FLF:\ b_2 = b_1(1 - N_{1,B}^{1/w}) + bN_{1,B}^{1/w}, \tag{2}$$

Table 1 **Dynamics of Customer Valuation Learning**

	Before release	Beginning of period 1	Beginning of period 2
CE	c_0	$c_1 = c_0$	Installed base at the end of period 1 $c_2 = c$ All other customers $c_2 = c_1 + N_1^{1/w} \cdot (c - c_1)$
FLF	a_0	$a_1 = 1$	$a_2 = 1$
	b_0	$b_1 = b_0 + \delta_b b$	Installed base at the end of period 1 $b_2 = b$ All other customers $b_2 = b_1 + N_{1,B}^{1/w} \cdot (b - b_1)$
S	c_0	Seeded customers $c_1 = c$ All other customers $c_1 = c_0$	Installed base at the end of period 1 $c_2 = c$ All other customers $c_2 = c_1 + (N_1 + k)^{1/w} \cdot (c - c_1)$

$$S:\ a_2 = a_1(1 - (N_1 + k)^{1/w}) + a(N_1 + k)^{1/w},\quad b_2 = b_1(1 - (N_1 + k)^{1/w}) + b(N_1 + k)^{1/w}, \tag{3}$$

where k represents the seeding ratio in the S model (introduced in §3.1). Here, w captures the strength of WOM effects. Define $c \triangleq a + b = 1 + b$ and $\alpha \triangleq (a_0 + b_0)/(a + b) = (\alpha_a + b\alpha_b)/(1 + b)$. Parameters c and α capture the true value of the full product (including premium features) and the deviation of the initial beliefs from c, respectively. Thus, consumers have an initial prior $c_0 \triangleq a_0 + b_0 = \alpha c$ on the value of c. We denote by $c_1 \triangleq a_1 + b_1$ and $c_2 \triangleq a_2 + b_2$ the consumer perceived full-product value at the beginning of periods 1 and 2, respectively. A complete picture of consumer valuation learning over time (putting together learning from own experience and WOM-based learning) is presented in Table 1. Although under *CE* and *S* the product is sold as a bundle, α_a and α_b may be distinct and Equations (1) and (3) capture the fact that the valuation updating for the whole product is based on the separate valuation updating processes for each of the two modules.

For a customer who did not adopt the product prior to the end of period 1, the belief updating process based on WOM at the beginning of period 2 reconciles *her* prior beliefs about the valuation with the signals she receives from *existing adopters*, disseminated in the form of online reviews. We assume that this valuation learning process follows a reduced-form weighted average parameterization, capturing a certain degree of inertia in belief updating due to the fact that the consumer has to internalize outside signals without the ability to completely verify their validity via own experience. Assuming a positive correlation between installed base and the number of reviews, our model accounts for the fact that sales, and, implicitly, consumer valuation learning and behavior, are affected by review *content and quality* and/or *number of reviews* (Chevalier and Mayzlin 2006, Dellarocas et al. 2007, Duan et al. 2008, Chen and Xie 2008, Ghose and

Ipeirotis 2011). There exists an extensive information processing literature suggesting that *content is not the sole determinant* of how people respond to a review. More precisely, the credibility and the ability of reviews to alter recipients' attitude and behavior have been demonstrated to depend often on the *attributes of the information source* (Kelman 1961, Chaiken 1980, Mackie et al. 1990, Chaiken and Maheswaran 1994, Petty et al. 1998, Menon and Blount 2003, Pornpitakpan 2004, Kang and Herr 2006, Forman et al. 2008). Firms acknowledge how the impact of reviews may be affected by the design of the online feedback platform and the contributed content. For example, as of November 2013, for a given product, Amazon lists the average review score, the number of reviews, text of each review, some information about the reviewer such as real name (indicated by the "real name" badge), city, and state (where made available by the reviewer—reviewers can opt out), whether the review originated from an actual buyer of the product, and a link to all other reviews contributed by that individual. Going beyond online reviews, in general, *confirmation* (or *confirmatory*) *bias*—a tendency to selectively favor new information that confirms initial beliefs—represents another widely documented factor that can induce individuals to exhibit stickiness in updating beliefs or strategies in spite of newly available information suggesting the optimality of an alternative course of action (Rabin and Schrag 1999, Jonas et al. 2001, Biyalogorsky et al. 2006). Based on all this rich extant literature, in our model, we use parameter w to quantify the degree of credibility and persuasiveness of online reviews in the given market, i.e., the weight that customers place on reviews when adjusting their prior beliefs (in a different context, Wu et al. 2013 use a similar model where network size is replaced by project size or scope). Since we normalize market size to 1, when w is high, it takes very few reviews to induce customers to change their beliefs significantly. On the other hand, in a market where w is low, customers are more reserved in deviating from their prior beliefs based on WOM. Moreover, the extent of belief updating depends on how far off customers are in their initial priors. We assume that customers form w based on general market experience.

In the special case of $w = \infty$, our WOM-based learning model would capture scenarios of perfect learning in one shot. Basically such a setting would describe a market where all outside signals are always fully persuasive (regardless of the size of the installed base spreading WOM) and they induce customers to completely abandon their prior perceptions about the value of the product in favor of the new signal (without being able to test its validity).[4] Thus, our framework is quite general, being able to accommodate both perfect and imperfect learning via WOM.

Equations (1)–(3) can also be applied to the belief updating for modules acquired in period 1 (for free or for a fee) since WOM effects do not change the valuation if consumers already learned the true value of either module A or B at the beginning of period 1 (i.e., $a_1 = a$ and/or $b_1 = b$). Furthermore, the above WOM-based belief updating formulation can be generalized to markets of size m by replacing $\{N_1, N_{1,B}, N_1 + k\}$ with $\{N_1/m, N_{1,B}/m, (N_1 + km)/m\}$.

We conclude this section by highlighting a couple of differences between experience-based learning and WOM effects. First, regardless of whether consumers overestimate or underestimate the product value in period 1, WOM effects push their beliefs of the true value in the correct direction. WOM effects originate from informed customers who have already used the respective functionality and learned its value. However, indirect experience-based effects, whereby customers change their belief about value of module B after trying module A (*FLF* model), are not based on actual use of module B and manifest independently of outside influence. Thus, such effects may or may not bring the prior on B closer to the correct value. Consider a simple example where customers initially overestimate the value of A and underestimate the value of B. Then, after receiving A for free, they realize they overestimated the value of the basic module A (which may be due to bugs, interface, compatibility, or any other issue that may prevent the user from extracting the expected value from using module A) and they may assume that this is the case for B as well (which may not be true if B is not relying heavily on the problematic part of A), which would lead to a further discounting of the perceived value of B. Further discussion is included in §4.5.

Second, in our framework, WOM effects take longer to manifest (adoption in period 1 induces a valuation update that has impact only on period 2 sales) compared to learning from own experience. Learning from own experience is one of the steps in generating WOM effects—existing users first learn themselves the value of the product. Then, it may take them time until they get to post informative online reviews. Furthermore, it takes time for information (in this case information in the online reviews) to disseminate in the market (Dodson and Muller 1978, Kalish 1985, Mankiw and Reis 2002, Morris and Shin 2006, Niculescu et al. 2012).

[4] Jing (2011) advances a hybrid WOM-based learning model (termed "social learning" in that paper) that combines both perfect learning and the impact of installed base in period 1 on period 2 learning. More precisely, in period 2, as a result of social learning, uninformed customers update their perception to the real value with a probability proportional to the installed base (sN_1) in period 1 (or, with probability $(1 - sN_1)$ keep their beliefs unchanged). There, s represents the "intensity" of social learning. The underlying assumption in Jing (2011) is that, if customers react in any way to WOM, they do so by dropping their priors completely and fully adopting the recommended value in the reviews.

3.3. Further Assumptions

We consider a market structure with information asymmetry. As previously discussed, customers do not know a priori the precise benefits from using the product. Furthermore, we assume customers do not know the consumer type distribution but know their own type and w. On the other hand, consistent with the literature (e.g., Chen and Seshadri 2007, Dey et al. 2013), we assume that the firm has incomplete information about consumers in the sense that it knows the consumer type distribution but does not know the specific type of any individual consumer. Moreover, we assume that the firm has full information about parameters b, α_a, α_b, δ_b, w.

We consider a sequential game, where the firm moves first by announcing at the beginning of period 1 the sales model and the price vector (p_1, p_2) for the two periods. Similar to Choudhary (2007) and Zhang and Seidmann (2010), we consider *credible* price commitments that are followed through. For example, as mentioned by the first reference, money-back guarantees can be used as one way to induce commitment credibility. For clarity, we reemphasize that the price is for a perpetual license (until obsolescence). Thus, first-period adopters get a *two-period* license for price p_1 and continue to use the product in the second period without any additional fees, whereas second-period adopters get a *one-period* license for price p_2. Once the prices have been announced, customers make the purchase decision in period 1 considering also the price in period 2 (i.e., the incentive compatibility (IC) constraint is taken into consideration). If they decide not to purchase, they can reevaluate that decision at the beginning of period 2, after WOM-based learning, in the context of one period of remaining useful life for the product.

We further assume that the firm and the consumers have different abilities to understand and process the information in the market. While firms pay close attention to the market and dedicate substantial effort in understanding the environment in which they commercialize their products (sometimes acquiring additional market intelligence from third-party information brokers), customers often times have been documented to exhibit cognitive limitations and, thus, *bounded rationality* in their strategies (Simon 1972, Rubinstein 1997, Kahneman 2003). Many factors have been shown to induce boundedly rational behavior of agents in various settings. In this paper, following Rubinstein (1993), Osborne and Rubinstein (1998), and Spiegler (2006), we consider consumer bounded rationality induced by the *complexity* inherent in processing in depth the available market information. Given the present information structure, it would be a very complex task for a customer to (i) start with a prior distribution on the infinitely many possible consumer type distributions and another prior on the per-period value of the product, and (ii) update these priors jointly based on the price disclosed by the firm, and, at the beginning of second period, sales after first period. Prior to the release of the product, the customers will form their initial opinion on the value of the product based on marketing campaigns ($\theta\alpha c$). This value can be considered an average over anticipated potential valuations. Customers retain a degree of consistency in their opinions at the beginning of period 1 in the sense that any information about how beliefs could change in the future is internalized in customers' current set of beliefs (if, for example, customers would anticipate that WOM would actually induce them with certainty to increase their per-period average valuation of the product there would be no reason to operate at the prior valuation levels). The true per-period product value is not changing in the future (there are no network effects at utility level) and, thus, future information via WOM will always be a signal about *the same* true value. Moreover, once the product is released on the market and the price is announced, customers do not try to reverse engineer the firm's strategy because of the complexity of such a process. Given that per-period value does not change over time and customers are unable to reverse engineer the firm's strategy to reveal the true value of the product or anticipate in which direction and by how much WOM effects will alter their valuation, we assume they always act according to present beliefs at the beginning of period 1. However, customers are not completely myopic in the sense that they do factor the prices associated with adoption in the first and second period when they consider their adoption decision in the first period (in other words both IC and incentive rationality (IR) constraints are factored into that decision).

Furthermore, the valuation updating at the beginning of period 2 via WOM (described in §3.2.2) represents a simple-enough heuristic approach through which consumers incorporate new information in their decision-making process for period 2 adoption while circumventing the complexity of considering all potential scenarios leading to that particular realization of adoption in period 1. The consideration of heuristics that depart from unbounded rationality but simplify decision making is common in models of consumer bounded rationality (Kahneman 2003, Spiegler 2006, Chen et al. 2010, Radner et al. 2013). In §3.2.2, we presented ample justification for the chosen WOM-based learning setup in this paper, connecting it to several extant theoretical frameworks describing the processing of information from external sources.

We point out that our model of bounded rationality is more justified in a setting like ours where the product does not exhibit network effects at utility level. If the true value of the product would change over time (for example because of network effects at utility level) and customers would know that, then they would have to anticipate somehow the impact of the installed base on the value of the product during periods 1 and 2.

4. Baseline Scenario—Constant Price

We start with a simple scenario where the firm commits to a constant price over time. Although arguably future generations of the same product introduce more features and improve quality, in this paper we focus on a single generation product where quality and functionality are more or less fixed. For tractability, we focus on the case of a digital goods market characterized by intermediate strength of WOM effects ($w = 1$) where reviews exhibit a moderate degree of persuasiveness. This, in turn, allows us to capture relevant trade-offs between experience-based learning and WOM effects. Similar linear parameterizations of valuation learning (but with respect to sample size and trial time) have also been used in Chellappa and Shivendu (2005) and Cheng and Liu (2012). We do relax this assumption in §§4.6 and 5 and show both numerically and analytically that insights continue to hold under different strengths of WOM effects.

Under constant pricing and bounded rationality, for any of the considered strategies, at the beginning of a given period, if some consumers did not previously adopt and their utility after belief updating is nonnegative, they immediately adopt (for period 1, IC is not binding since, for the same price, two-period consumption always dominates one-period consumption). This is consistent with the model in Chatterjee and Eliashberg (1990), whereby customers learn their valuation of the product via various market signals and adopt as soon as their expected utility based on current beliefs is nonnegative. In their model, some customers delay adoption in earlier periods, but that is because at those points in time they believe the adoption would yield negative utility. In our model, under constant price for a perpetual license, delay in adoption is based on similar considerations.

4.1. Equilibrium for *CE* Model

We first derive the equilibrium for the conventional *CE* business model. Given the bounded rationality assumptions in §3.3, consumers will never consider at the beginning of period 1 a strategy whereby to delay adoption until period 2 because at that point in time they perceive the benefits from two periods of usage at $2c_1\theta$ while benefits from one period of usage are perceived at $c_1\theta$. Therefore, the incentive compatibility constraint at the beginning of period 1 is automatically satisfied if the incentive rationality constraint is satisfied, and potential adopters consider solely between adopting at the beginning of period 1 and not adopting at all. Thus, in order to make profit, the firm only considers the feasible pricing strategies $0 < p < 2c_1$. Let θ_1 be the *marginal* (lowest) type consumer that adopts the software in period 1. Then $\theta_1 = p/(2c_1) = p/(2\alpha c)$. All consumers with type $\theta \geq \theta_1$ buy the software in the first period. Thus, $N_1 = 1 - \theta_1$. At the end of period 1, potential customers who have not adopted yet (i.e., those consumers with type $\theta < \theta_1$) update their estimate of c (more precisely, a and b) to c_2 (a_2 and b_2). A consumer with type $\theta < \theta_1$ would adopt the software in period 2 and use it only for a single period of time if and only if $c_2\theta \geq p$. Let θ_2 be the *marginal* existing adopter at the end of period 2, regardless of when she adopted the product. Then

$$\theta_2 = \begin{cases} \theta_1, & \text{if } \dfrac{p}{c_2} \geq \theta_1, \\[2ex] \dfrac{p}{c_2}, & \text{otherwise.} \end{cases} \tag{4}$$

The installed base at the end of period 2 is $N_2 = 1 - \theta_2$. The following lemma captures the conditions for adoption in period 2.

Lemma 1. *Under CE, for any feasible price $p \in (0, 2c_1)$, adoption extends to the second period ($0 \leq \theta_2 < \theta_1$) if and only if $c_2 > 2c_1$.*

Lemma 1 illustrates the trade-off that potential customers are facing at the beginning of the second period if they did not adopt in the first period. On one hand, for the same price p, they can only benefit one period from using the product. On the other hand, their priors on c have been adjusted from c_1 to c_2, and, as a result, their WTP for one period of software use has changed. One interesting aspect captured in Lemma 1 is that consumers must at least double their priors on c as a result of WOM effects in order to be willing to adopt in period 2. Weak adoption in period 1 does not provide potential customers with enough market feedback for them to consider a change in their adoption decision. Moreover, if consumers overestimated c in the beginning ($\alpha > 1$), then the updating at the end of period 1 lowers their priors ($c_2 < c_1$) because of calibration in the correct direction induced by WOM. In such a case, there is no adoption in period 2 because consumers' WTP decreased. Replacing c_2 in Lemma 1 leads to the following result:

Lemma 2. *Under CE, adoption extends to the second period if and only if consumers initially significantly underestimate the value of the software ($\alpha < \frac{1}{2}$) and the price is sufficiently low ($p < 2c\alpha(1-2\alpha)/(1-\alpha)$).*

Lemma 1 explores period 2 adoption conditions in terms of updated estimate c_2, which in turn depends on period 1 adoption and, implicitly, on price. Lemma 2 advances our understanding of the adoption dynamics by illustrating that adoption never occurs in period 2 when consumer priors are high, price is high, or both.

We have fully characterized above the consumers' equilibrium response to any given price. Next, we solve the firm's profit maximization problem:

$$\max_{0<p<2c\alpha} p \cdot N_2. \tag{5}$$

This is a nontrivial problem since different pairs $\{\alpha, p\}$ induce different adoption patterns, as illustrated in Lemma 2. Final installed base N_2 is a function of p that depends on whether there is adoption in the second period or not. To simplify the notation, we first define function f on $(0,\infty)$ as follows:

$$f(x) \triangleq \begin{cases} \dfrac{2x}{(\sqrt{1+x}+\sqrt{2x})^2}, & x \in (0, 13-4\sqrt{10}],\\[2mm] \dfrac{x}{2}, & 13-4\sqrt{10} < x. \end{cases} \tag{6}$$

It can be easily shown that f is strictly increasing, bijective (thus, having an inverse that is also increasing), and differentiable everywhere except at $13-4\sqrt{10}$. We describe in the next proposition the firm's optimal pricing strategy.

Proposition 1. *Under CE and constant pricing, the firm's optimal pricing strategy and profit are*:

	$0<\alpha<13-4\sqrt{10}$	$13-4\sqrt{10}\le\alpha$
p^*_{CE}	$\dfrac{2c\alpha}{1-\alpha}\left(1-\sqrt{\dfrac{2\alpha}{1+\alpha}}\right)$	$c\alpha$
π^*_{CE}	$cf(\alpha)$	$cf(\alpha)$
Paid adoption	*In both periods*	*Only in period 1*

Figure 1 illustrates the firm's pricing strategy, profit, and induced adoption patterns.[5] In equilibrium, if consumer priors are low (i.e., $\alpha < 13-4\sqrt{10} \approx 0.351$), then the firm will price the product such that adoption occurs in two periods. In this case, even though later adoption involves shorter usage time, some of the customers make the purchase in period 2 after updating their priors at the end of period 1 because of WOM effects. When α is very small, the firm will employ a price that is low enough to generate a very small but significant enough mass of adopters in period 1 ($\lim_{\alpha\downarrow 0} N_1 = 0$) who, in turn, fuel WOM effects that make most potential customers adopt in period 2 ($\lim_{\alpha\downarrow 0} N_2 = 1$). When consumer WTP is higher, the firm will charge more (p^*_{CE} is increasing in α) and there will be more period 1 adopters (N_1 is increasing in α) but fewer period 2 adopters ($N_2 - N_1$ is decreasing in α) and fewer overall adopters (N_2 is decreasing in α). Thus, as α grows, the firm exploits more the initial higher WTP of consumers and less period 2 adoption as a result of WOM effects on valuation learning. In this region, N_2 is always greater than $\frac{1}{2}$. On the other hand, N_1 is lower than $\frac{1}{2}$ when $\alpha < \sqrt{5}-2$, and higher than $\frac{1}{2}$ when $\alpha \in [\sqrt{5}-2, 13-4\sqrt{10}]$. If $\alpha \ge 13-4\sqrt{10}$, the firm boosts the price linearly with α and exploits high WTP *solely* in period 1 such that $N_1 = N_2 = \frac{1}{2}$. In particular, if consumers overestimate the true value of the software ($\alpha > 1$), the firm exploits this perception bias immediately through a higher price since it understands that period 1 consumption is critical in capturing the highest profit given that in period 2 WOM about lower-than-expected software quality will spread and will reduce per-period consumer WTP.

4.2. Equilibrium for Freemium Model

The derivation of the equilibrium solution for the *FLF* model is similar to that for the *CE* model by substituting $c \to b$ and $\alpha \to \alpha_b + \delta_b$.[6]

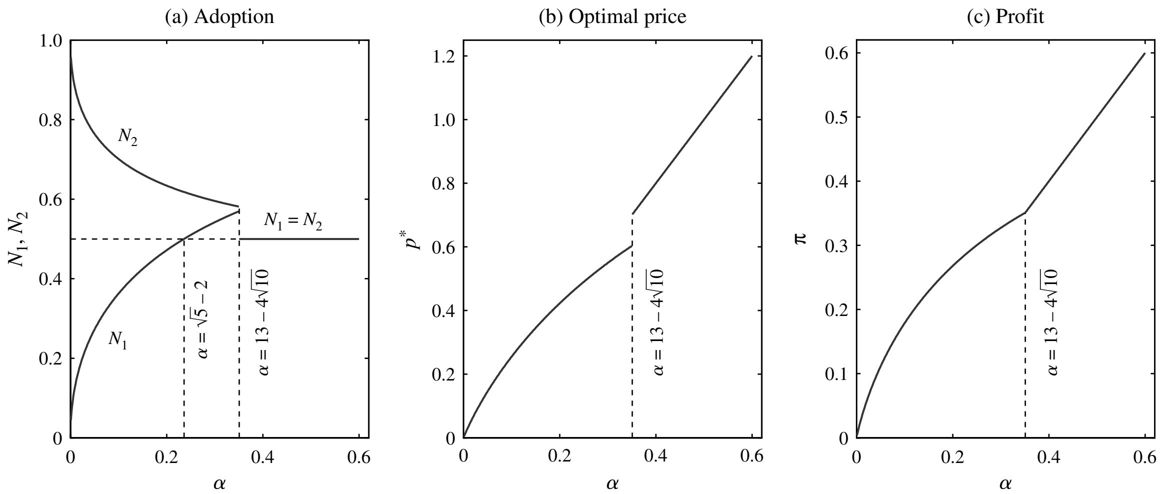
	$0<\alpha_b+\delta_b<$ $13-4\sqrt{10}$	$13-4\sqrt{10}\le$ $\alpha_b+\delta_b$
p^*_{FLF}	$\dfrac{2(\alpha_b+\delta_b)b}{(1-\alpha_b-\delta_b)} \cdot \left(1-\sqrt{\dfrac{2(\alpha_b+\delta_b)}{1+\alpha_b+\delta_b}}\right)$	$b(\alpha_b+\delta_b)$
π^*_{FLF}	$bf(\alpha_b+\delta_b)$	$bf(\alpha_b+\delta_b)$
Paid adoption	In both periods	Only in period 1

The adoption pattern is similar to the one depicted in Figure 1, but with the horizontal axis describing $\alpha_b+\delta_b$ instead of α. Also, in this case, N_1 and N_2 should be replaced by $N_{1,B}$ and $N_{2,B}$. When $\alpha_b+\delta_b \ge 13-4\sqrt{10}$, the firm's optimal pricing under *FLF* results in exactly half of the market being covered for the premium module.

[5] When $\alpha = 13-4\sqrt{10}$, the firm can obtain the same profit in two ways. At a high price, it can restrict adoption to period 1, whereas at a lower price it will induce adoption in both periods (with $N_2 - N_1 > 0$). All else being equal, we assume that the firm prefers to get the revenue earlier and keep the price high (perhaps as a statement of the quality of the product). Similar considerations are made in the remaining part of the paper.

[6] Under *CE*, the firm sells product *A*&*B* with true per-period value factor c and deviation factor for initial beliefs $c_1/c = \alpha$. Under *FLF*, after indirect learning because of cross-module synergies, the firm sells product B with true per-period value factor b and deviation factor for initial beliefs $b_1/b = \alpha_b + \delta_b$.

Figure 1 ***CE* Equilibrium** ($c = 2$)



4.3. Equilibrium for Seeding Model

In our framework, consumer learning depends on the existing network size but not on the types of customers who adopted before. If the firm had visibility into individual consumer types, then it would seed consumers who otherwise would not buy the product, instead of giving away the software to any high-type consumers. However, in a more realistic setting, the firm has limited knowledge of consumer types. For that reason, we consider *uniform seeding*, whereby k% of the addressable market receives the software for free in period 1 (with $k \in [0, 1)$) and each type is uniformly represented in the product giveaway pool.

In this model, in period 1, in addition to seeding, the firm also sells the software. WOM from the seeded community gets around at the beginning of period 2 even when nobody else purchases the product in the first period. For paid adoption to start in period 1, it must be the case that $p < 2c\alpha$ (because the product has two periods of residual life remaining). Alternatively, if paid adoption occurs only in period 2, then it must be the case that $2c\alpha \le p < c\alpha + k(c - c\alpha)$ because customers update their valuation based on WOM effects induced by the seeds but face only one period of residual product life. Therefore, the feasible price condition becomes

$$0 < p < \max\{2c\alpha, c[\alpha + k(1-\alpha)]\}. \tag{7}$$

When constraint (7) is satisfied, adoption occurs in either period 1, period 2, or both. We note that $2c\alpha < c[\alpha + k(1-\alpha)]$ if and only if $\alpha < k/(1+k)$. Therefore, for prices in the feasible range, we have the following:

$$\theta_1 = \begin{cases} \dfrac{p}{2c\alpha}, & \text{if } \dfrac{k}{1+k} \le \alpha \text{ and } p < 2c\alpha, \\ \dfrac{p}{2c\alpha}, & \text{if } 0 < \alpha < \dfrac{k}{1+k} \text{ and } p < 2c\alpha, \\ 1, & \text{if } 0 < \alpha < \dfrac{k}{1+k} \text{ and} \\ & \quad 2c\alpha \le p < c[\alpha + (1-\alpha)k], \end{cases} \tag{8}$$

where $\theta_1 = 1$ indicates that no paid adoption occurs in period 1. Then, at the end of period 1, we have $N_1 = (1-k)(1-\theta_1)$ paying customers. It immediately follows that $c_2 = c\alpha + (N_1 + k)c(1-\alpha) = c - c(1-k)(1-\alpha)\theta_1$.

Solving for the optimal price p_S^* and seeding ratio k_S^* is a hard problem with a nontrivial proof but it yields a surprising and elegant equilibrium solution.

Proposition 2. *Under S and constant pricing, the firm's optimal seeding, pricing, and profit are*:

	$0 < \alpha < \underline{\alpha}$	$\underline{\alpha} \le \alpha < 13 - 4\sqrt{10}$	$13 - 4\sqrt{10} \le \alpha$
k_S^*	$\dfrac{1-2\alpha}{2-2\alpha}$	0	0
p_S^*	$\dfrac{c}{4}$	p_{CE}^*	p_{CE}^*
π_S^*	$\dfrac{c}{16(1-\alpha)}$	π_{CE}^*	π_{CE}^*
Paid adoption	*Only in period 2*	*In both periods*	*Only in period 1*

where p_{CE}^* and π_{CE}^* are given in Proposition 1, *and $\underline{\alpha} \approx 0.065$ is the unique solution to the equation $g(\alpha) = 0$ over the interval $(0, 13 - 4\sqrt{10})$, with $g(\alpha) = 1/(16(1-\alpha)) - 2\alpha/(\sqrt{1+\alpha} + \sqrt{2\alpha})^2$.*

Existence and uniqueness of $\underline{\alpha}$ is proved in online supplement B in auxiliary Lemma B4. We note that seeding is optimal only for very low priors, i.e., when consumers severely underestimate the value of the full product. Once $\alpha \geq \underline{\alpha}$, the firm can generate strong-enough WOM effects via sales in period 1 and it will follow a *CE* strategy since that way it is not losing any sales to high-type customers. Under optimal seeding and pricing, in period 1 we have either seeding or paid adoption (sales), but never both. In §4.6 we further explore the sensitivity with respect to w of the optimal seeding ratio and cutoff α value where *S* defaults to *CE*.

We emphasize that the above result applies to uniform seeding when costumers exert similar peer influencing abilities. In such cases, uniform seeding involves potential sales cannibalization as some of the high-type customers end up with the product for free. If information acquisition is not costly, the firm might prefer to learn the customer types and implement a more targeted seeding strategy. Similar to other studies, in our paper, we do not incorporate such trade-offs and restrict our attention to markets where the firm cannot have access to detailed customer information at a low cost.

4.4. Comparative Statics

The following result summarizes firm's optimal choice among the three business models.

Proposition 3. *From a profit perspective, for any feasible set* $\{\alpha_a, \alpha_b, \delta_b, b\}$, *if the firm can choose among CE, FLF, and S business models, the following hold true*:

(a) *If* $\underline{\alpha} \leq \alpha$, *then*

(i) *if* $\delta_b \leq f^{-1}(((b+1)/b)f(\alpha)) - \alpha_b$ *then CE is the dominant strategy;*

(ii) *otherwise, FLF is the dominant strategy.*

(b) *If* $\alpha \in (0, \underline{\alpha})$, *then*

(i) *if* $\delta_b \leq f^{-1}((b+1)/(16b(1-\alpha))) - \alpha_b$ *then S is the dominant strategy;*

(ii) *otherwise, FLF is the dominant strategy,*

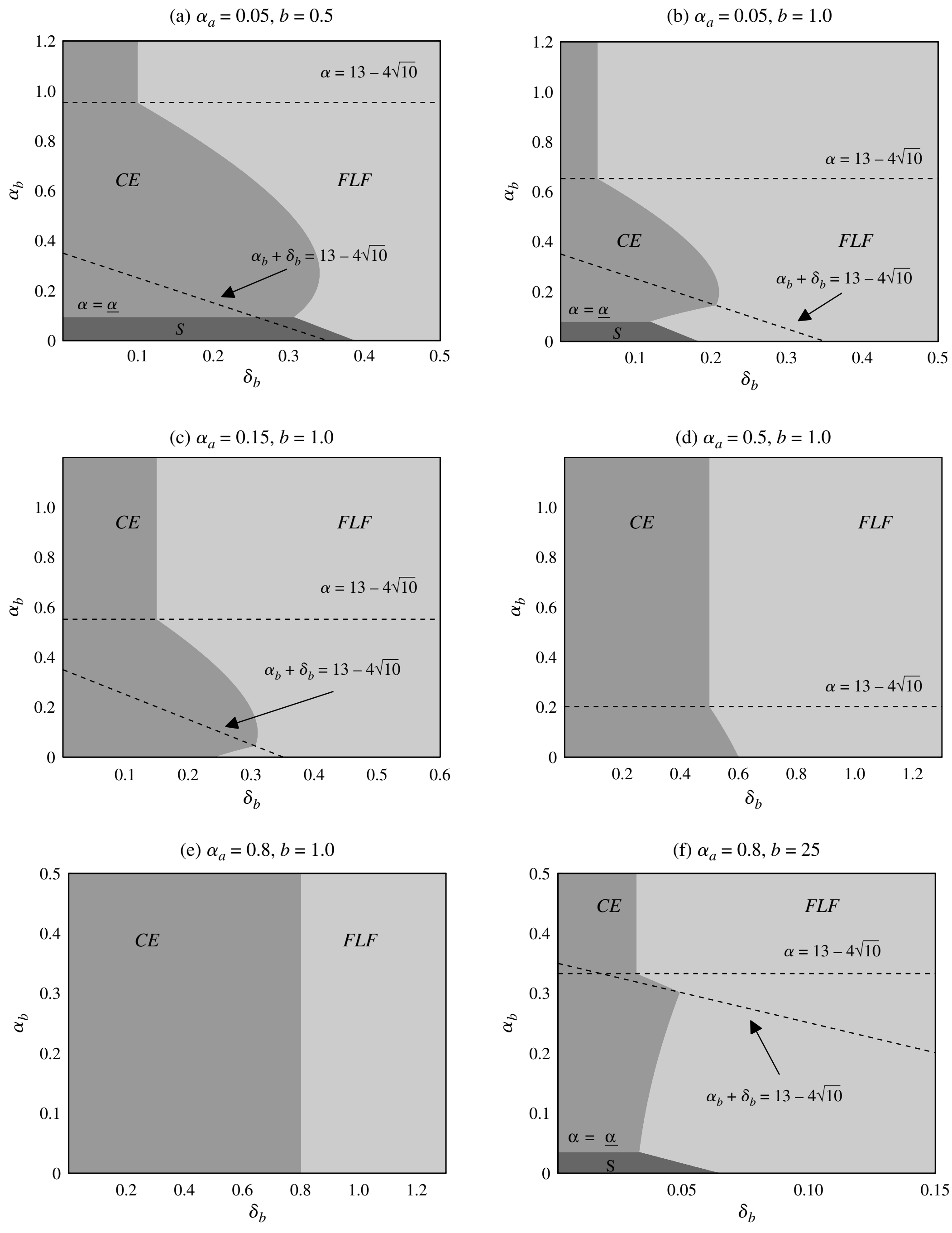
where $\alpha = \alpha_a/(b+1) + b\alpha_b/(b+1)$ *and* $\underline{\alpha} \approx 0.065$ *were defined in Proposition* 2.

Figure 2 captures the essence of the above result for various parameter sets. As expected, for very large values of δ_b, *FLF* will be the dominant strategy. Similarly for very small values of δ_b, *FLF* will be dominated by the other two strategies since giving away module *A* does not induce a major positive updating of customers' valuations for module *B*. In such cases, customers only consider the bundled product and their aggregate valuation prior ($c_1 = \alpha c$) determines whether *S* or *CE* are optimal. In panels (a), (b), and (f) of Figure 2 we recognize the cutoff point $\alpha = \underline{\alpha}$ between *CE* and *S* optimality regions that was introduced in Proposition 2 (for a given α_a, as in each panel of Figure 2, any level of α_b corresponds to a particular level of α). As discussed in §4.3, if customers severely underestimate the value of the bundle, the firm will seed a significant portion of the market and price the product at a high level such that sales occur only in period 2. Otherwise, if customer priors are not too low, the firm will not resort to seeding. For a given α_b, the threshold δ_b value beyond which *FLF* becomes optimal is unique and depends on the difference in value between the two modules because *FLF* involves a trade-off between lost sales on *A* and increased WTP for *B*. Many game apps are offered under a freemium model, with limited content/functionality (e.g., levels, resolution, etc.) available for free under a "lite" version and additional fees charged for premium content/functionality. By trying a couple of free levels, users learn about the interface and concept of the game, which may induce them to update significantly their valuation for additional content.

What is interesting and nontrivial is the irregular nature of the *boundary* between the optimality region for *FLF* and the optimality regions for the other two strategies. In particular, for a given cross-module synergy effect δ_b, moving on the vertical axis, an increase in the initial consumer priors for the valuation of module *B* (i.e., an increase in α_b) can induce the firm to switch from *S* to *FLF* to *CE* and, ultimately, back to *FLF* (see panels (a), (b), (f), and, to some extent, (c) of Figure 2). Thus, there are regions of the parameter space where, all other things equal, *FLF* is optimal when the priors on premium functionality are either relatively low or high, *but not in between*. As α_b increases, the boundary between the optimality region for *FLF* and the optimality regions for the other strategies can move left, right, or remain unchanged with respect to δ_b level, depending on the market conditions that characterize a particular region in the feasible space. This is due to the fact that a change in the initial valuation for module *B* simultaneously impacts the outcome of every strategy (as it affects the valuation for the bundle as well). However, the magnitude of the impact may differ from one strategy to another. Thus, when choosing their prerelease advertising strategy, it is important for managers to understand that different marketing campaigns (which lead to the formation of different priors on the value of the two modules) might need to be paired in nontrivial ways with different commercialization strategies. Failing to account for such effects may lead to suboptimal decisions on the firm's side. We elaborate below in more detail on some of the mechanics behind the above-mentioned irregularities in the boundary for the *FLF* optimality region.

When α is sufficiently large ($\alpha \geq 13 - 4\sqrt{10}$), for any given cross-module synergy effect δ_b, an increase in the initial consumer priors for module *B* will never induce the firm to change its strategy (the firm sticks to

Figure 2 Firm's Optimal Strategies



either *CE* or *FLF*). In this region, it can be shown that the firm prefers *CE* iff $\delta_b \le \alpha_a/b$ (see Corollary B1 in online supplement B). When $\delta_b > \alpha_a/b$, it immediately follows that $\alpha_b + \delta_b > (b+1)/b \times \alpha > \alpha > 13 - 4\sqrt{10}$. In this case, as discussed in §§4.1 and 4.2, under both *CE* and *FLF*, customers value the bundle and module *B* (after experience-based learning) high enough such that the firm prices high in period 1 and does not try to capitalize on WOM (no period 2 sales). Under each strategy, exactly half of the customers buy the non-free offered product (bundle or module *B*). For each paying customer, the difference in WTP under *CE* and *FLF* is $2\theta(\alpha(1+b) - (\alpha_b + \delta_b)b) = 2\theta(\alpha_a - \delta_b b)$. Thus, the firm can charge each of the high types more under *FLF* than under *CE* when $\delta_b > \alpha_a/b$ in spite of offering less functionality in the non-free product. In the opposite direction, when $\delta_b \le \alpha_a/b$, experience-based learning can never lead to a recovery of lost sales under freemium, regardless of whether sales occur in one or two periods under *FLF*.

We next focus on small enough values of α (where, under *CE*, the firm would try to capitalize on WOM effects and induce sales of the bundle in both periods) and small to intermediate ranges of δ_b (where cross-module synergies do not lead to either negligible or excessive boosts in the valuation of module *B*). As can be seen from panels (a)–(c), and (f) in Figure 2, when α_a is low or b is high (which leads to $\alpha = \alpha_a/(b+1) + b\alpha_b/(b+1)$ to be relatively small for small values of α_b) in general there will exist a region $(\delta_{b,L}, \delta_{b,H})$ where, ceteris paribus, as α_b increases, it is possible for the firm to switch from *S* to *FLF*, *FLF* to *CE*, and/or *CE* to *FLF*. An increase in α_b can induce the firm to switch from *S* to *FLF*, but not vice versa. It can be shown that $(\partial \pi_S^*/\partial \alpha_b)(\alpha) < (\partial \pi_{FLF}^*/\partial \alpha_b)(b, \alpha_b, \delta_b)$ when α is small. When customers have a very low initial prior on the bundle, a small increase in the consumers' prior on module *B* (that still keeps $\alpha \le \underline{\alpha}$ so that *S* dominates *CE*) benefits more *FLF* than *S*. Thus, as α_b increases, a lower δ_b is sufficient to give *FLF* an advantage. Note still that in such regions, a significant δ_b value (relative to α_a and α_b) is necessary in order for *FLF* to dominate *S*.

Perhaps the most interesting type of switch induced by an increase in α_b is from *FLF* to *CE* and then back to *FLF*. As α_b increases, both π_{CE}^* and π_{FLF}^* increase, but they do so at *different* rates. Given that $\alpha = \alpha_a/(b+1) + b\alpha_b/(b+1)$, $\pi_{CE}^* = (1+b)f(\alpha)$, and $\pi_{FLF}^* = bf(\alpha_b + \delta_b)$, it immediately follows that $\partial \pi_{CE}^*/\partial \alpha_b = bf'(\alpha)$ while $\partial \pi_{FLF}^*/\partial \alpha_b = bf'(\alpha_b + \delta_b)$. Thus, in order to compare the profit increase in *CE* and *FLF*, it is necessary to compare $f'(\alpha)$ and $f'(\alpha_b + \delta_b)$. However, f' is not monotone, where defined (note that f' is not defined at $13 - 4\sqrt{10}$, in which case it is necessary to use left and right derivatives). Note that panel (c) of Figure 1 captures the behavior of $\pi_{CE}^* = cf(\alpha)$ and, thus, it can be used to understand the behavior of function f as c is just a scaling factor. More precisely, f is increasing, concave on $(0, 13 - 4\sqrt{10})$ and linear afterwards. Thus, for each of *FLF* and *CE*, in the region where the firm takes advantage of WOM (when there is adoption in period 2, i.e., $\alpha < 13 - 4\sqrt{10}$ or $\alpha_b + \delta_b < 13 - 4\sqrt{10}$), an increase in the value of the initial prior has *diminishing returns* on profit. This occurs because, as α increases, the firm relies less on period 2 sales and focuses more on higher valuation customers who adopt in period 1 based on their initial priors. As discussed at the end of §4.1, whereas period 1 sales (N_1) increase, period 2 sales ($N_2 - N_1$) and overall sales (N_2) decrease in α. Moreover, $f'(0+) > \frac{1}{2}$, $f_L'(13-4\sqrt{10}) < \frac{1}{2}$, $f_R'(13-4\sqrt{10}) = \frac{1}{2}$, $f'(x) = \frac{1}{2}$ for all $x > 13 - 4\sqrt{10}$, where f_L' and f_R' denote left and right derivatives. As α approaches $13 - 4\sqrt{10}$ (region below that line) and $\delta_b + \alpha_b > 13 - 4\sqrt{10}$, we can see in panels (a)–(c) of Figure 2 that *FLF* benefits more from an increase in α_b and, consequently, a lower δ_b is necessary for it to dominate *CE*. This pushes the boundary between optimality regions for *FLF* and *CE* to the left. However, for smaller α values (smaller α_b values), the impact of an increase in α_b can be stronger for *CE* than for *FLF* as α can lie closer to 0 while $\alpha_b + \delta_b$ can be further away from it. In such cases, when consumers' initial prior for *B* increases, a stronger cross-module synergy effect is necessary to make *FLF* the preferred strategy. This, in turn, pushes the boundary between optimality regions for *FLF* and *CE* to the right.

We point out that the firm takes advantage of WOM effects (period 2 sales) when *S* is optimal, *CE* is optimal and $\alpha < 13 - 4\sqrt{10}$, or *FLF* is optimal and $\alpha_b + \delta_b < 13 - 4\sqrt{10}$. Otherwise, firm tries to maximize period 1 sales and kills period 2 sales because customers already believe that one or both modules hold substantial value and WOM effects do not have the potential to generate enough consumer valuation upgrade in such regions. As in the case of switches between strategies, we can also identify very interesting and nontrivial switches between regions where the firm capitalizes on WOM effects (sales in period 2) and regions where it does not. For example, in panel (a) of Figure 2, if we consider $\delta_b \approx 0.32$, an increase in α_b gradually induces the firm to switch from capitalizing on WOM (under *S*) to selling only in period 1 (under *FLF*), then again to capitalizing on WOM (under *CE*), and then back to selling only in period 1 (under *FLF*). Thus, in addition to the previously mentioned changes in business model, a prerelease advertising campaign targeting premium features also has the potential to fundamentally alter the firm's reliance on WOM in nonintuitive ways.

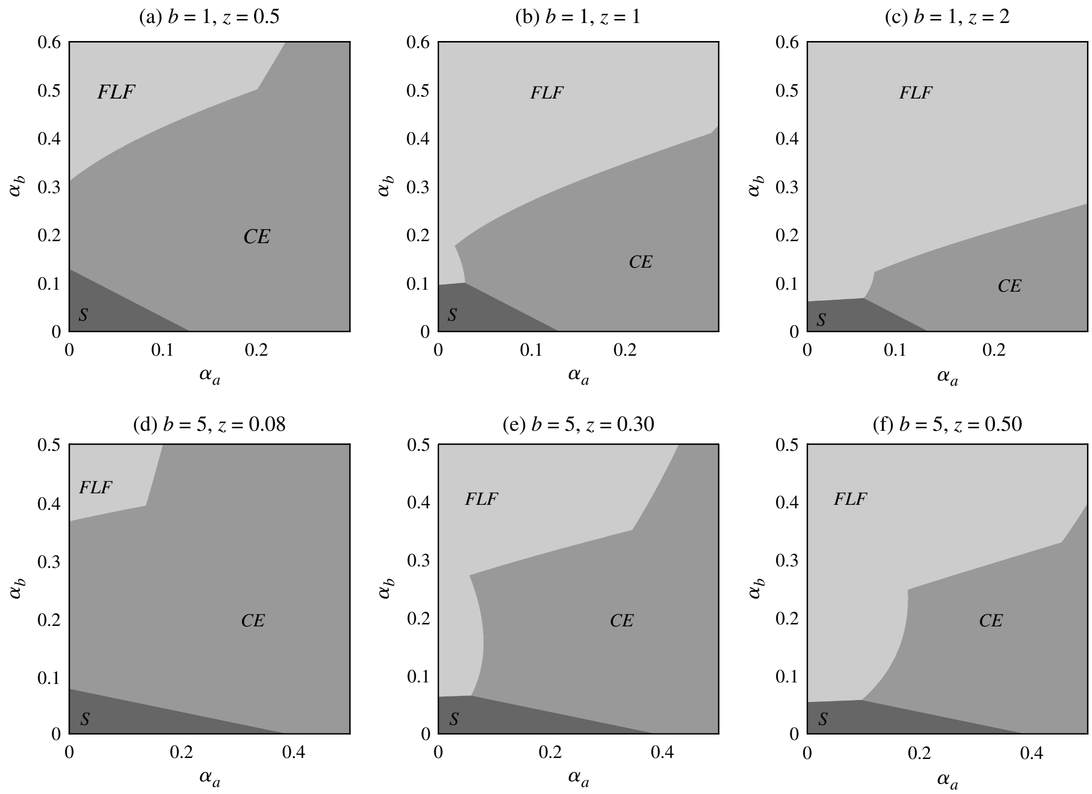
4.5. Parametric δ_b

Previously, the comparative statics results in §4.4 explore all possible parameter regions. However, dependencies between parameters may reduce the feasible regions and the range of potential outcomes. In particular, the strength of the experience-based learning due to cross module synergies may depend on some of the other parameters. Under *FLF*, once customers are given *A* for free, they understand by how much their initial valuation was off compared to the actual value of that functionality. Furthermore, they understand whether they initially overvalued or undervalued module *A*. As customers realize the direction and magnitude of the initial estimation error on *A*, one possible reaction could be to adjust the prior on *B* in the same direction through a somewhat correlated correction. One possible parameterization that captures this dynamic is

$$\delta_b(\alpha_a, \alpha_b) = \alpha_b \cdot \max\{-1, z(1-\alpha_a)\}. \tag{9}$$

If α_a is fairly close to 1, then a consumer might have little incentive to distrust her initial priors and abandon them. As a result, she might not update much the estimate of the value of module *B*. However, if

Figure 3 **Firm's Optimal Strategies When δ_b Is Parameterized According to Equation** (9)



the prior on the value of module A was dramatically adjusted and there are strong synergies between the two modules, the consumer might be more inclined to reevaluate her beliefs on the value of B and update her estimate in the same direction as the correction on the beliefs on A. Parameter z incorporates the customers' perceived degree to which the modules are interrelated once A is revealed. If the value of A was initially overestimated, customers adjust downward the estimate of B. The lower bound on the adjustment indicates that in the worst-case scenario customers would consider module B worthless. Note also that the above parameterization imposes an implicit upper bound $\alpha_b z$ on δ_b. Consumers do not observe δ_b but can compute valuation adjustment $\delta_b b = (\alpha_b b) \cdot \max\{-1, z(1-\alpha_a)\}$ because they know their initial prior $b_1 = \alpha_b b$ on the product, and α_a was revealed via directly experiencing A.

As can be observed in Figure 3, in such a setting, S is always optimal for small α_a and α_b because in such regions δ_b is implicitly small and both *FLF* and *CE* end up being dominated. As α_b increases, whether the firm moves from S to *FLF* or to *CE* depends on z. The stronger the perceived connection between the two modules is (i.e., higher z), the more advantage *FLF* gets (in panels (a)–(c) and (d)–(f) of Figure 3 we show changes in optimality regions based on increases in z when $b = 1$ and $b = 5$, respectively). Also, *FLF* emerges as the dominant strategy when α_b is very large. On the other hand, as α_a gets stronger, *FLF* starts losing its edge because of two effects: (i) customers are willing to pay more for module A and the opportunity cost of lost sales increases when the firm gives it away for free, and (ii) the increase in the WTP for module B has a lower magnitude. Regardless of α_a, as long as α_b is very small, *FLF* cannot be dominant as δ_b will not be high enough to beat S (when α_a is low) or *CE* (when α_a is high). Thus, if the current parameterization of δ_b is true, the bottom right region of all panels in Figure 2 is infeasible. Furthermore, when customers overestimate the value of A, then *FLF* will never be optimal as δ_b will be negative. When customers underestimate the value of A, for high enough values of α_b, *FLF* will always dominate the other strategies. Thus, the top left part of panels in Figure 2 is infeasible when $\alpha_a \leq 1$ because high α_b translates into high δ_b.

We remind the reader that the interesting dynamics captured in panels (a)–(c), (f) of Figure 2, where a switch from *FLF* to *CE* and back could be preferred as α_b increases, occur in §4.4 for low to moderate α and α_b, moderate δ_b, and low $\alpha_a/(b+1)$. In the context of the current parameterization, when α_a is low and b is not too high, in order for S not to dominate, we need a moderate α_b (such that α is not too low). In conjunction with it, a moderate δ_b can be achieved only when z is neither too high nor too low. This can be seen by visual examination of panels (a)–(c) of Figure 3.

Similar considerations hold when b is high, but the level of z (δ_b) required in small ranges of α_a for *FLF* to gain an edge over *CE* is smaller, as can be seen from panels (d)–(f) of Figure 3. Compared to the switches described in §4.4, it is possible to have an even more complex switching between optimality regions when the priors on premium functionality valuation increase under the current parameterization. In particular, in panel (e) of Figure 3 we see that when $\alpha_a \approx 0.07$ an increase in α_b can induce a switch from *S* to *CE* to *FLF* to *CE*, and back to *FLF*. Moreover, we see that promotional prerelease advertising campaigns that boost both α_a and α_b (i.e., campaigns covering all the product functionality) can induce similar switches in strategy from *S* to *FLF* to *CE* to *FLF*. We can see in panels (b), (c), (e), (f) that it is possible to draw straight but *sloped* lines (even through origin) that cross the regions in the aforementioned sequence. In panels (e) and (f) we can clearly see that such switches can actually end up under *CE* as long as α_a keeps increasing fast enough alongside α_b.

Alternative parameterizations can also be considered. For example, z may be parameterized as $z(b) = \bar{z}/b$. Customers form $z(b)$ after observing *A* but do not know either $\bar{z}$ or b. As more functionality is packed in module *B*, it may become harder to see how it is all connected to the few features in module *A*. As such, if b is large, even though customers do not know this value, they may be more reluctant to deviate from their original prior on *B* following their experience with *A*.

4.6. Different Levels of w

The above analysis considers an intermediate level of WOM effects, quantified by $w = 1$. In such a case, customers are not drastically deviating from their priors when just a few reviews are available but are also not overly sticky in their behavior once a high volume of reviews conflict with their initial priors. However, it is important to understand in general how w will affect firm's optimal strategy. Some of the insights are illustrated in Figure 4.

When WOM effects are very weak (small w), *S* is a less appealing strategy. Giving the product away for free to some of the customers has little effect on the other customers. Moreover, seeding uniformly implies that some of the high types are always seeded, which represents a drawback under weak WOM effects and not-too-low priors. Thus, α, α_b, and δ_b would have to be very small for *S* to be the dominant strategy.

As w grows, WOM effects are stronger and customers can alter their attitude and behavior significantly after being exposed to a lower number of reviews. As such, when α, α_b, and δ_b are relatively small, the area where seeding is optimal extends with w. Under both *CE* and *FLF*, benefits of stronger WOM effects can only be harnessed via *paid* adoption in period 1. When customers significantly undervalue the modules in the very beginning, the firm would have to keep the price low under either one of these two strategies, which would yield lower profits compared to the profit under *S*. The optimality region for *S* expands because, under higher w, impactful WOM effects can be induced via a smaller uniformly seeded pool, resulting in less cannibalized demand at the top tier. In turn, this leaves available a larger pool of higher type customers that the firm can sell to in period 2 at a higher price, once these customers have updated their product valuation.

The effects discussed in the above paragraph are illustrated in Table 2. We consider the same setup as in Figure 4 ($\alpha_a = 0.05$ and $b = 1$) where a low-enough prior on the value of module *A* leads to seeding being optimal when module *B* is undervalued and, also, offering *A* for free does not reveal much about the value of *B*. We denote by $\alpha_{S,\,CE}(w)$ the threshold prior on the value of the entire product above that *S* always defaults to *CE*. When $w = 1$, then $\alpha_{S,\,CE}(1) = \underline{\alpha} \approx 0.065$, as discussed in §4.3. For any given w, $\alpha_{S,\,CE}(w)$ is independent of δ_b and *S* dominates *CE* if and only if $\alpha < \alpha_{S,\,CE}(w)$ or, equivalently, $\alpha_b < \alpha_{b,\,S,\,CE}(w) \triangleq ((b+1) \cdot \alpha_{S,\,CE}(w) - \alpha_a)/b$. We denote $k^*_{S,\,\min}(w) \triangleq \min\{k^*_S(\alpha_b, w) \mid \alpha_b \in (0, \alpha_{b,\,S,\,CE}(w))\}$

Figure 4 **Firm's Optimal Strategies for Various Levels of Strength of WOM Effects** ($\alpha_a = 0.05$, $b = 1$)

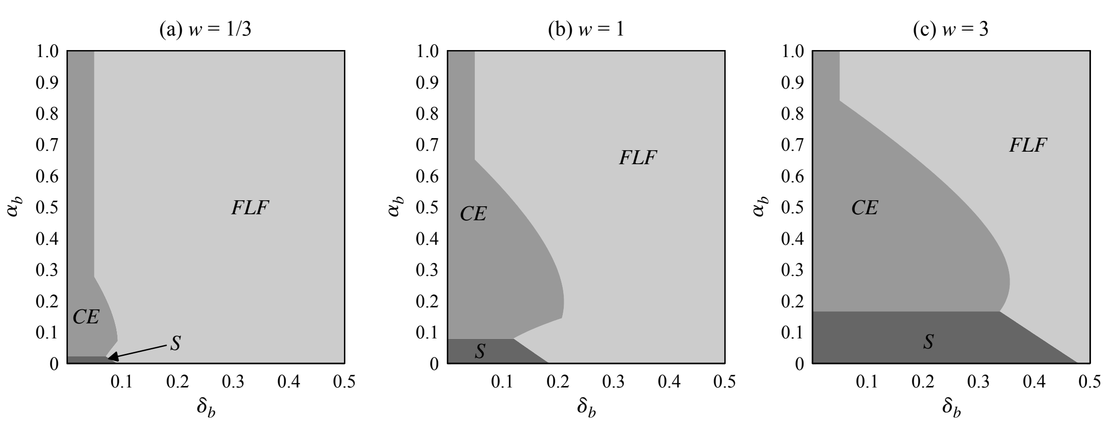


Table 2 Impact of Stronger WOM Effects on S Strategy ($\alpha_a = 0.05$ and $b = 1$)

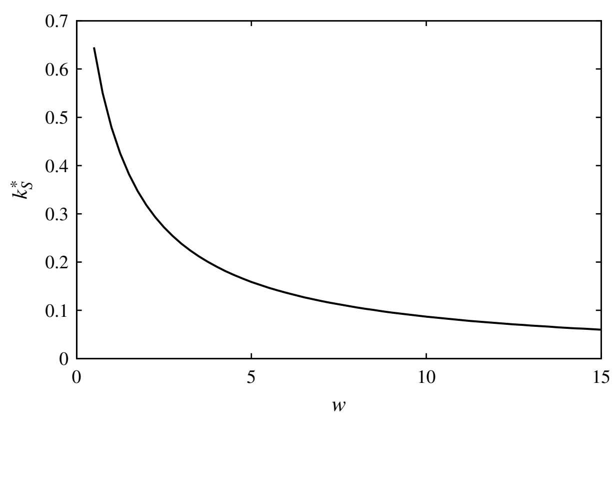
w	1/3	1	3	5	10	∞
$\alpha_{S,CE}(w)$	0.0360	0.0647	0.1083	0.1303	0.1582	0.25
$\alpha_{b,S,CE}(w)$	0.0221	0.0794	0.1665	0.2107	0.2665	0.45
$k^*_{S,\min}(w)$	0.7326	0.4655	0.2172	0.1407	0.0744	0^+
$k^*_{S,\max}(w)$	0.7382	0.4872	0.2425	0.1617	0.0883	0^+

and $k^*_{S,\max}(w) \triangleq \max\{k^*_S(\alpha_b, w) \mid \alpha_b \in (0, \alpha_{b,S,CE}(w))\}$. We point out that, for a given set $\{w, \alpha_a, b\}$, *all* $k^*_S(w)$ values in the interval $(k^*_{S,\min}(w), k^*_{S,\max}(w))$ are feasible. This is because when δ_b is very small, *FLF* will not be optimal, and thus, S emerges as the dominant strategy in *all* scenarios where $\alpha_b < \alpha_{b,S,CE}(w)$.

As can be seen from Table 2, $\alpha_{S,CE}(w)$ and $\alpha_{b,S,CE}(w)$ are increasing in w, consistent with Figure 4, confirming that the area in which seeding is optimal expands as w increases. Moreover, both $k^*_{S,\min}(w)$ and $k^*_{S,\max}(w)$ are decreasing in w, supporting the argument that in regions where S is the dominant strategy, as WOM effects get stronger, the firm should seed less. Zooming in on a given α_b level, Figure 5 captures the monotonicity of the optimal seeding rate k^*_S with respect to w for a particular parameter set $\{\alpha_a, \alpha_b, b\}$.

The α threshold for *CE* above which the firm abandons pricing that induces two-period adoption (i.e., the region where the firm exploits WOM effects) and focuses solely on first period adoption ($\alpha = 13 - 4\sqrt{10}$ when $w = 1$) also shifts upward as w increases (similarly for *FLF*). However, this threshold never goes beyond $\frac{1}{2}$. When $\alpha > \frac{1}{2}$, regardless of w, WOM effects cannot more than double consumers' perception about the value of one period of use and, thus, consumers who passed on period 1 adoption do not change their mind in period 2. The upward shift of the threshold as a result of higher w is due to the fact that the firm can charge more in period 1 and, at the same time, get stronger increase in WTP of customers in period 2.

Figure 5 Evolution of k^*_S with Respect to w ($\alpha_a = 0.05$, $\alpha_b = 0.03$, $b = 1$)



Thus, as WOM effects grow stronger, the firm can harness the benefits of WOM in a wider parameter region altogether.

In regions of the parameter space where *CE* relies on WOM but *FLF* does not (low α but high $\alpha_b + \delta_b$), we see from Figure 4 that an increase in the strength of WOM effects tends to move the indifference curve between *CE* and *FLF* strategies to the right because *CE* will capitalize on WOM (two-period adoption) while *FLF* may benefit less or not at all from WOM (depending on whether in the respective parameter region *FLF* will start to rely or not on WOM as well).

We next explore in more detail the special case of extreme WOM effects ($w = \infty$). In such markets, for firms to take *full* advantage of WOM, all they need is a *negligible but positive* installed base at the end of period 1. Customers lack any stickiness in behavior and immediately discard their prior beliefs in favor of the opinion of previous adopters as soon as they are exposed to the first opinion coming from an adopter. In other words, the degree of persuasiveness of reviews is extreme in inducing customers to adjust their beliefs. Although this may not be realistic (as discussed in §3.2.2), it represents an informative benchmark case.

Proposition 4. *When the strength of WOM effects is extreme ($w = \infty$), from a profit perspective, for any feasible set $\{\alpha_a, \alpha_b, \delta_b, b\}$, if the firm can choose among CE, FLF, and S business models, the following hold true*:

(a) *if $\delta_b \geq \max\{(b+1)/(2b) - \alpha_b, \alpha_a/b\}$, then FLF is the dominating strategy*;

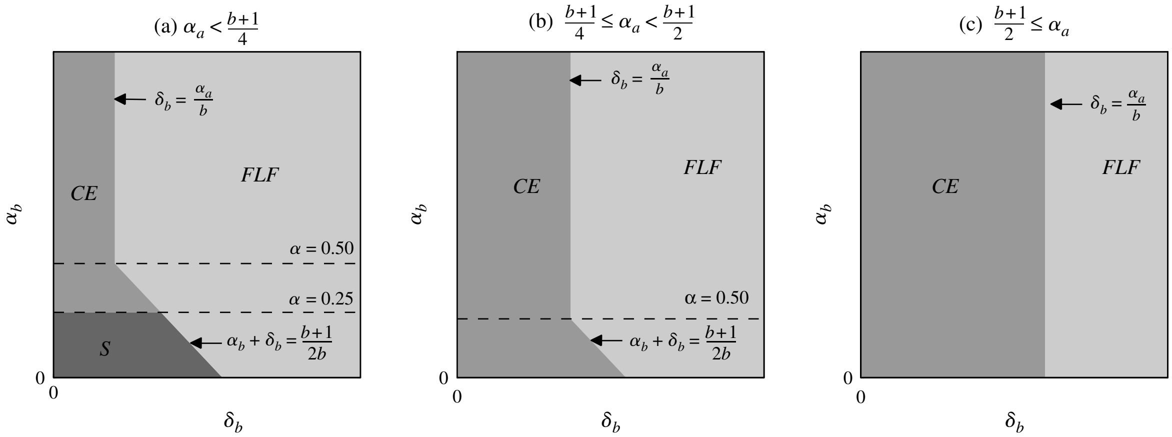
(b) *if $0 < \alpha_a \leq (b+1)/4$, $\alpha_b \leq (b+1)/(4b) - \alpha_a/b$, and $\delta_b \leq (b+1)/(2b) - \alpha_b$, then S is the dominating strategy and $k^*_S = 0^+$, where 0^+ denotes a negligible positive quantity*;

(c) *otherwise CE is the dominating strategy.*

Firm's strategies under extreme WOM effects are illustrated in Figure 6. First, we note that it is possible for each strategy to be dominant. Even though under $w = \infty$ it takes very few adopters in period 1 for everyone to know the true value of the product at the beginning of period 2, when customers significantly undervalue the product and cross-module synergies are weak, it is only under S that WOM effects can be started without compromising to a low price (under *CE* and *FLF*, the firm is constrained by the need to price low to get WOM effects started). In such regions, the firm will seed a negligible but nonzero mass of customers ($k^*_S = 0^+$), thus getting everyone perfectly informed in period 2.

For high w, when α_a and α_b are low but δ_b is moderate, we can see from panel (a) of Figure 6 that it is still possible for the firm to switch among the three strategies as α_b increases (from S to *CE* to *FLF*). What changes compared to lower levels of w is that in such cases, as α_b increases, it is never optimal to switch from *FLF* to *CE* (unlike in the three panels in Figure 4).

Figure 6 Firm's Optimal Strategies Under Extreme WOM Effects ($w = \infty$)



This is due to the fact that profit under *CE* is piecewise linear in the regions where it dominates *S*. Moreover, we point out that when *FLF* is optimal, it never relies on WOM (all sales are in period 1). Optimal pricing strategies under *CE* and *S* are derived in online supplement B in Propositions B1 and B2.

4.7. Impact of the Number of Periods

The above analysis focuses exclusively on two-period scenarios, where WOM effects can only manifest once. In this section we discuss the robustness of our results with respect to the number of periods, T, assuming WOM effects take place at the end of *each* period. Sensitivity with respect to T can be considered in two distinct contexts:

(i) Constant module value over lifetime ($2a\theta$ and $2b\theta$ for modules A and B for a customer of type θ). This setup corresponds to a constant life span of the product but allows for WOM effects to manifest at different time intervals. In each period, customers can derive $2a\theta/T$ and $2b\theta/T$ value from each of the modules. Comparing different scenarios of T is equivalent to considering *similar* products performing in *different* markets with different patterns of information dissemination via WOM.

(ii) Constant per-period module value ($a\theta$ for module A and $b\theta$ for module B). In this case, comparing different scenarios of T is equivalent to considering *different* products (same per-period value but different life spans) performing in *similar* markets with respect to information dissemination via WOM.

The two contexts can be shown to be equivalent in terms of chosen market strategy. For a given T, as long as b/a is constant, we obtain the same split of the feasible space into regions based on which strategy is optimal whether the two modules exhibit per-period value parameters $\{2a/T, 2b/T\}$ or $\{a, b\}$. However, optimal prices and profits will be different in the two contexts as they depend on the residual value of the product to the consumers (basically the prices and profits will maintain the proportionality between per-period module values under the two contexts, i.e., a factor of $2/T$).

Insights are very similar to those in the previous section, where sensitivity with respect to w was considered. More periods allow the firm to extract more value out of WOM effects because of their reiteration after each period. When customers undervalue each module a lot and δ_b is small, S emerges as the optimal strategy (since under seeding the firm can activate WOM effects without compromising on price). As δ_b increases, *FLF* starts getting an edge. Otherwise, *CE* will dominate. As T increases, the region in which S is the optimal strategy expands (as illustrated in Figure C1 in online supplement C) while the optimal seeding ratio k_S^* decreases. A detailed discussion would follow similar arguments as in §4.6 and, hence, is omitted for brevity. Nevertheless, we point out that, different from the discussion in §4.6, customers have the opportunity to make a purchase decision at T points rather than just two points along the timeline. Hence, in the case of many periods, customers may learn a lot about the value of the product before half of the product life is gone and hence have a chance to make a purchase decision when there is a lot more residual value remaining in the product. This, in turn, may increase consumer WTP at relatively early stages (compared to the setup with $T = 2$), and makes a difference in terms of profits especially in the case of seeding where the firm does not need to use price to activate WOM. Hence, boundaries between optimality regions will look a little less smooth compared to the previous case but insights remain the same qualitatively.

In the special case of extremely fast information dissemination (T approaches ∞) in context (i), S dominates *CE* when $\alpha \in (0, \frac{1}{2})$. Thus, we will have a much larger region where S will be the optimal strategy compared to the case when $w = \infty$ and $T = 2$ (in which case $\alpha_{S,CE} = 0.25$, as illustrated in Table 2 in §4.6). This is precisely because under $T = \infty$, once WOM effects are activated, customers learn almost instantaneously the value of the product, and, unlike before, can actually make a decision when the remaining residual value of the product is almost identical to the full life-time value of the product. The above arguments are formalized in Propositions C1–C3 and Figure C2 in online supplement C.

5. Dynamic Pricing

Previously we considered scenarios where the firm maintained the same price during both periods. In this section, we relax this constraint and assume the price can change over time. We retain the credible price commitment framework and the sequence described in §3.3. Because under the assumed bounded rationality setup customers do not anticipate at the beginning of period 1 that their priors will change in the future, under constant pricing no consumer would purposely delay adoption and the IC constraint is never binding. However, under dynamic pricing, a substantial difference in prices between the two periods may induce customers to delay adoption and forgo one period of use even if they do not expect a change in their beliefs about the per-period value of the product. As such, IC constraint may be binding for the marginal customer in period 1.

Under dynamic pricing, the firm is able to take better advantage of WOM effects. Under constant pricing for *CE* and *FLF*, when valuation parameters α and $\alpha_b + \delta_b$ are low, the firm is forced to price low in the first period to spur some adoption but is later bound by that pricing decision also in period 2, which may drastically limit its future revenues. Consequently, in such regions, *S* dominates since its associated price was not necessarily intended to spur period 1 paid adoption because WOM effects can be propagated by the seeded customers. By contrast, under dynamic pricing, the firm can adjust its second period price and better capitalize, if it chooses to do so, on the updated WTP of the customers under all models.

We first examine the *CE* strategy (and, implicitly, the *FLF* strategy). We start by presenting some general properties of the optimal strategy under various strengths of WOM effects:

Proposition 5. *Under CE with dynamic pricing and commitment, for any positive strength of WOM effects* ($w > 0$), *the optimal strategy satisfies the following properties*:

(a) *When* $\alpha > 1$, *the firm will choose prices such that IR is binding for period* 1 *and there is no adoption in period* 2. *More precisely,* $p^*_{CE,1} = c\alpha$ *and* $p^*_{CE,2} \geq p^*_{CE,1}/2$.

(b) *When* $\alpha \in [\frac{1}{2}, 1]$, *the firm will offer prices such that IC and IR constraints are simultaneously binding in period* 1 *and there is period* 2 *adoption. Moreover, in such cases,* $p^*_{CE,2} = p^*_{CE,1}/2$.

(c) *There exists* $\alpha^\dagger < \frac{1}{2}$ *such that, when* $\alpha < \alpha^\dagger$, *IC constraint is no longer binding and IR constraints are binding in both periods. In this region, optimal prices default to the prices under myopic behavior and no commitment.*

(d) *There exists* $\alpha^\ddagger \leq \alpha^\dagger$ *such that penetration pricing* ($p^*_{CE,1} < p^*_{CE,2}$) *is optimal whenever* $\alpha < \alpha^\ddagger$.

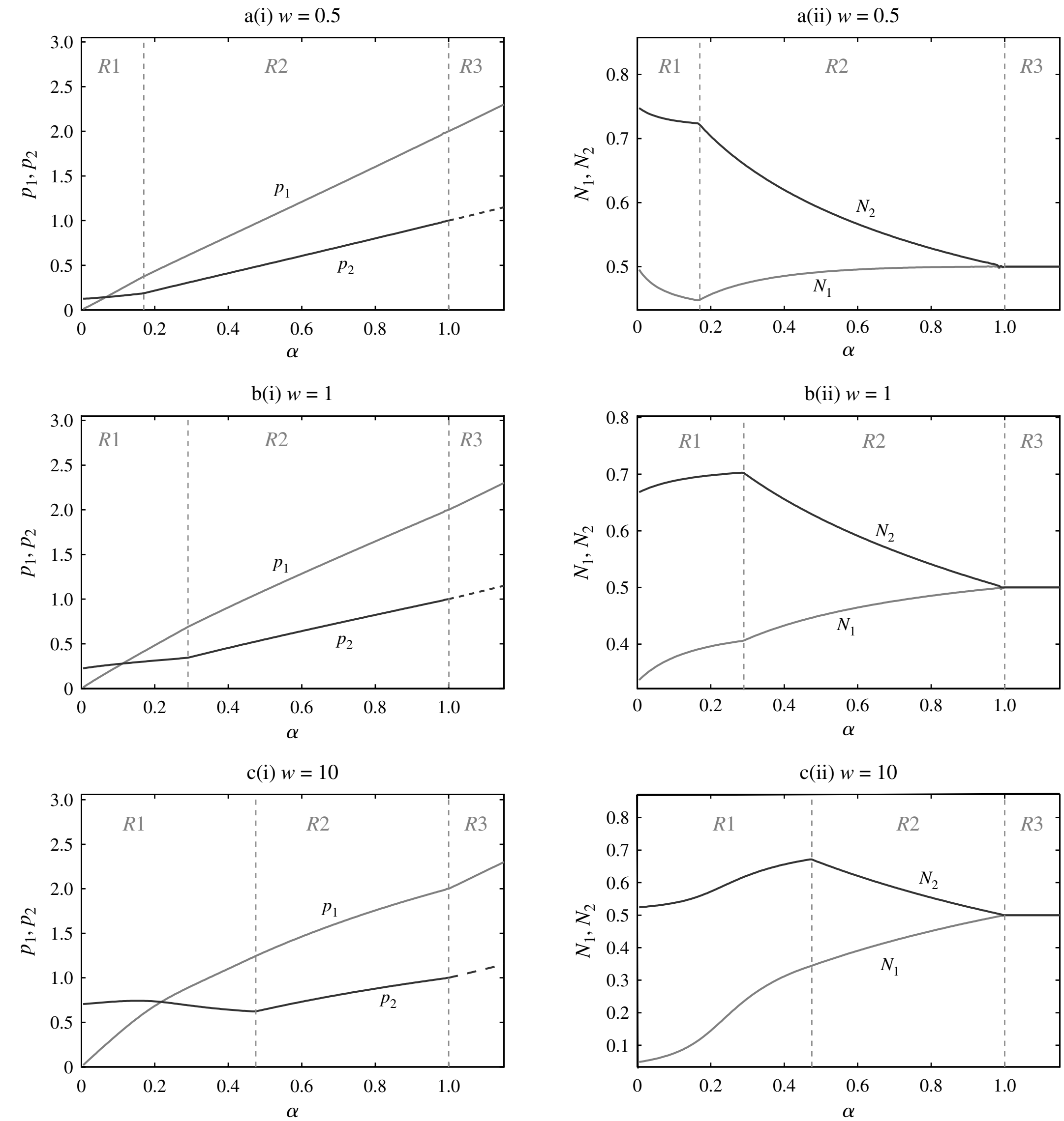
We illustrate in Figure 7 the optimal pricing strategy and the induced adoption for various levels of strength of WOM effects. For simplicity, we drop the model subscript (*CE*) in the following discussion and in the figure. As discussed in Proposition 5(a), when $\alpha > 1$ (region *R*3), regardless of w, it is suboptimal to have any sales in period 2 and prices are set such that customers would behave as if myopic[7] in period 1. This is achieved by choosing $p_1(\alpha) = c\alpha$ and $p_2 \geq p_1/2$ when $\alpha > 1$. For simplicity, we assume the firm sets $p_2(\alpha) = p_1(\alpha)/2$ when $\alpha > 1$ since the outcome is the same for all $p_2(\alpha)$ above that level. In Figure 7, p_2 function is marked as a dotted line after $\alpha > 1$ to illustrate the fact that this is just one of the many ways to achieve optimality.

When α is very small (region *R*1), there is potential for customers to learn a lot via WOM, and this potential is higher for stronger WOM effects (larger w). As such, in this region, the firm prices in period 1 just to get adoption started and kick off WOM effects. Then, it capitalizes on these effects by charging a high price in period 2 which, for low enough α, is even higher than p_1 as illustrated in part (d) of Proposition 5. Penetration pricing has been shown to be optimal under various conditions where adoption is influenced in one way or another by the installed base (e.g., see Kalish 1983) and it is often observed in practice when a new product is introduced in the market through a promotional campaign that involves discounts for a limited period of time. Because of the big jump in WTP induced by WOM effects, the optimal p_2 is high and, as such, IC constraints are not binding (customers in period 1, before WOM takes effect, would consider period 2 price too high for their perceived value of one period of product use). Thus, optimal revenues in periods 1 and 2 are similar to revenues under myopic consumer behavior. Part (c) of Proposition 5 captures the essence of this outcome.

Both region *R*1 (where consumers' decisions match the myopic scenario, before IC becomes binding in period 1) and the subregion of *R*1 where penetration pricing is optimal (i.e., $p_2 > p_1$) extend as WOM effects grow stronger. We see that, as w increases, the firm tends to increase p_1 and yet get enough adoption in period 1 to achieve a strong increase in WTP in period 2 such that it finds optimal to charge $p_2 > p_1/2$ (which

[7] In this framework, customers are considered myopic if they do not incorporate p_2 in their purchase decision in period 1.

Figure 7 ***CE* Equilibrium Under Dynamic Pricing for Various Levels of Strength of WOM Effects When $c = 2$**



Notes. In region *R*1, IR constraints are binding in both periods and IC constraint is not binding in period 1. In region *R*2, IC and IR constraints are simultaneously binding in period 1 and IR constraint is binding in period 2. In region *R*3, IR constraint is binding in period 1, IC constraint is not binding in period 1, and p_2 is set such that no adoption occurs in period 2.

can be shown to prevent IC from being binding) for a wider range of α values. Eventually, though, benefits of WOM decrease as α increases because consumer priors are closer to the real value and the potential boost in WTP is smaller. In this case, period 2 prices are half of period 1 prices, and IC becomes binding in period 1.

It can be easily seen that in low-α region *R*1 the firm prices the software such that in period 2 exactly half of the remaining market adopts. In this region, although p_1 tends to always increase, p_2 may deviate from this pattern depending on the strength of WOM effects. In particular, close to the point where it is optimal to have IC constraint binding for the marginal adopter in period 1 (close to *R*2 region), if WOM effects are very strong, it is possible for p_2 to be decreasing, as illustrated in panel (c(i)) of Figure 7. Under such conditions, on one hand, as α increases, period 1 has higher revenue potential for the firm whereas the benefit of WOM decreases. As long as α is very low, as N_1 increases in α, due to WOM, period 2 valuation gets updated in a stronger way and customers are willing to pay more in period 2. As such, p_2 is initially increasing in α. On the other hand, since $c_2 = c_1 + N_1^{1/w}(c - c_1)$, if N_1 and w are high enough, $N_1^{1/w}$ is very close to 1. Thus, if N_1 is not too small, changes in N_1 do not lead to large upward changes in consumers' valuation in period 2 (as learning is almost perfect). However, as α increases, consumer WTP increases significantly in period 1, not only due to α but also to the fact that the customers are facing two periods of usage. Hence, as α increases, there is less incentive for the firm to push adoption in period 2. We see a gradual increase in N_1, (as can be seen in $R1$ region in panel (c(ii)) of Figure 7), which captures these effects. Beyond a certain threshold for α, coupled with the optimal N_1 increase, there is less of a market potential available for period 2 (those customers who did not adopt in period 1) and consumers' valuation changes only very little in period 2. As a result, since in period 2 the firm aims at covering half of the remaining market (when IC is not binding), it must reduce p_2. For WOM effects of intermediate strength, as illustrated in panels (b(i)) and (b(ii)) of Figure 7, WTP increases substantially in period 2 even for intermediate levels of α as N_1 increases but is not yet very large. As such, the firm chooses to increase p_2 for higher α as it faces a smaller market but where customers are willing to pay considerably more than in period 1. At the other end of the spectrum, for weak WOM effects, as illustrated in panels (a(i)) and (a(ii)) of Figure 7, still period 2 is valuable when customers initially severely undervalue the product. However, as α increases, there is significantly less potential to increase WTP via WOM. As such, pricing low in period 1 to boost adoption would yield decreasing benefits to the firm as α increases. Consequently, once α increases, the firm engages in a more aggressive period 1 pricing, which leads to a smaller but more profitable installed base in period 1 (for low w, N_1 is decreasing in α in region $R1$).

Once α moves into intermediate ranges (region $R2$), period 2 is less valuable under all models since there is only limited potential to boost customer WTP via WOM. Then, both p_1 and p_2 are pushed higher, IC is binding, and adoption in period 2, $N_2 - N_1$, is gradually phased out in favor of period 1 adoption. In such ranges, since learning via WOM is limited, price would have to drop significantly in period 2 to encourage significant adoption. That, in turn, would push a lot of customers to wait in period 1 (unless p_1 is also small), which would be suboptimal given the increased customer WTP at the beginning of period 1. Moreover, as can be seen, overall adoption at the end of period 2, N_2 is decreasing. Overall, larger WTP would allow the firm to be more aggressive in its pricing strategies and focus on a smaller but more profitable top tier of customers, most of whom adopt in period 1. Compared to the constant pricing scenario, where the firm prefers not to have period 2 sales when $\alpha > 13 - 4\sqrt{10}$ (see §4.1), under dynamic pricing there are always period 2 sales as long as $\alpha < 1$. In other words, under dynamic pricing, as long as there is an opportunity to increase the per-period valuation of customers via WOM, the firm will capitalize on WOM effects.

It is very hard to derive the optimal pricing under CE for general w. However, we are able to do it for the case of WOM effects of intermediate strength ($w = 1$—panel (b(i)) in Figure 7), as described in the next result.

Proposition 6. *Under CE with dynamic pricing and commitment, when $w = 1$, the optimal strategy is as follows*:

(a) *Region $R1$: $0 < \alpha < \frac{1}{2\sqrt{3}}$. Then*

$$p^*_{CE,1} = \frac{2c\alpha(\sqrt{40\alpha^2 + 8\alpha + 1} - 8\alpha + 1)}{3(1-\alpha)},$$

$$p^*_{CE,2} = \frac{p^*_{CE,1}(2c\alpha - p^*_{CE,1}(1-\alpha))}{8c\alpha^2},$$

$$\pi^*_{CE} = \frac{p^*_{CE,1}}{32c^2\alpha^3}(32c^2\alpha^3 + p^*_{CE,1}2c\alpha(1-8\alpha) - p^{*2}_{CE,1}(1-\alpha)),$$

and paid adoption occurs in both periods.

(b) *Region $R2$: $\frac{1}{2\sqrt{3}} \le \alpha \le 1$. Then*

$$p^*_{CE,1} = \tilde{\xi}(c, \alpha), \quad p^*_{CE,2} = \frac{p^*_{CE,1}}{2},$$

$$\pi^*_{CE} = \frac{p^*_{CE,1}(2c\alpha - p^*_{CE,1})(4c\alpha - p^*_{CE,1}(1-\alpha))}{4c\alpha(2c\alpha - p^*_{CE,1}(1-\alpha))},$$

where $\tilde{\xi}(c, \alpha)$ is the unique solution to the equation $\phi_{c,\alpha}(\xi) = 0$ over the interval $[0, 2c\alpha]$, with $\phi_{c,\alpha}(\xi) = -(1-\alpha)^2\xi^3 + c\alpha(6-\alpha)(1-\alpha)\xi^2 - 4c^2(3-\alpha)\alpha^2\xi + 8c^3\alpha^3$. Paid adoption occurs in both periods.

(c) *Region $R3$: $\alpha > 1$. Then $p^*_{CE,1} = c\alpha$, $p^*_{CE,2} \ge c\alpha/2$, $\pi^*_{CE} = c\alpha/2$, and paid adoption occurs only in period 1.*

Following exactly the same transformation as in §4.2, based on the optimal pricing under CE we can derive the optimal pricing under FLF. The seeding strategy is very similar to the one under constant pricing and is given by the following result.

Proposition 7. *Under S with dynamic pricing and commitment, when $w = 1$, the firm's optimal seeding ratio, pricing, and profit are*:

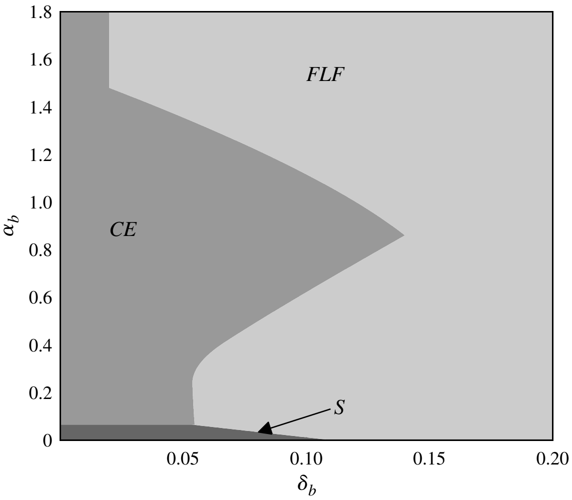
	$0 < \alpha < \underline{\hat{\alpha}}$	$\underline{\hat{\alpha}} \le \alpha < 1$	$1 \le \alpha$
k^*_S	$\dfrac{1-2\alpha}{2-2\alpha}$	0	0
$p^*_{S,1}$	$2c\alpha$	$p^*_{CE,1}$	$p^*_{CE,1}$
$p^*_{S,2}$	$\dfrac{c}{4}$	$p^*_{CE,2}$	$p^*_{CE,2}$
π^*_S	$\dfrac{c}{16(1-\alpha)}$	π^*_{CE}	π^*_{CE}
Paid adoption	*Only in period 2*	*In both periods*	*Only in period 1*

where $p^*_{CE,1}$, $p^*_{CE,2}$, π^*_{CE} are given in Proposition 6 and $\underline{\hat{\alpha}} \approx 0.0557$ is the unique solution to equation $\hat{g}(\alpha) = 0$ over the interval $[0, \frac{1}{16})$, with $\hat{g}(\alpha) = 1/(16(1-\alpha)) - (1 + 12\alpha - 132\alpha^2 - 224\alpha^3 + (1 + 8\alpha + 40\alpha^2)^{1.5})/(54(1-\alpha)^2)$.

We note that where *S* is optimal, the seeding ratio and period 2 price are identical to the ones identified in Proposition 2 under constant pricing. Also, similar to the constant pricing scenario, seeding and period 1 sales never occur in tandem under optimal seeding. When α is low, WOM effects induced by the seeds can significantly boost consumer WTP in period 2. High-type unseeded consumers will be willing to pay in period 2 more than in period 1 even though the remaining life of the product is shorter. As such, the firm blocks any sales in period 1 by pricing the product high enough to deter adoption prior to WOM effects. Moreover, the α region where it is optimal to seed shrinks in size compared to the one under constant pricing ($0 < \hat{\underline{\alpha}} < \underline{\alpha}$) because dynamic pricing adds some flexibility to *CE*. We note that the added flexibility is still not enough for *CE* to (weakly) dominate *S* everywhere. Although the firm can now adjust the price in the second period under *CE* (which it could not do under constant pricing), *S* still has one advantage in regions where α is small. Under *CE*, when initial consumer valuation is low, the firm tries to capitalize on WOM effects. As such, at least in period 1, the firm will price low to induce some adoption and start WOM. However, once the high-type customers perceive the price as reasonable, they *all* adopt in period 1 under *CE*. As such, even though WOM induces an increase in WTP for the other customers, the highest type customers have all adopted in period 1 and, thus, in period 2, the firm has access to a segment with a lower maximum WTP. Under uniform seeding, the firm will give the product to only *k*% of every type segment and thus, can still sell to some of the very high valuation customers in period 2. This is possible because WOM gets started by the seeded customers even when no additional customers purchase the product in period 1.

Given that under dynamic pricing, when using *CE* (or *FLF*) models, the firm can take better advantage of WOM effects compared to the scenario of constant pricing, the question is whether the insights in §4 continue to hold. Because of the complex expressions (sometimes in implicit form) for the profits under *CE* and *FLF*, we omit formalizing a result equivalent to Proposition 3. Conceptually, similar insights apply. For every strategy, there is a region in the parameter space where it is optimal. There is a cutoff α value that separates the regions where *S* and *CE* are optimal. Moreover, ceteris paribus, an increase in δ_b causes a unique switch from *S* to *FLF* or from *CE* to *FLF* (large δ_b favors *FLF*). When δ_b is very small, there is no incentive to give away A for free, and *FLF* is dominated. When α and $\alpha_b + \delta_b$ are large (beyond 1), *S* is dominated and, as discussed above, we only have period 1 adoption for *FLF* and *CE* with profits matching the ones under constant pricing. In such regions of high priors, similar to the constant pricing case, the cutoff

Figure 8 **Firm's Optimal Strategies Under Dynamic Pricing** ($w = 1$, $\alpha_a = 0.04$, $b = 2$)



line $\delta_b = \alpha_a/b$ separates the region where *CE* is optimal from the region where *FLF* is optimal. Furthermore, the interesting nontrivial shape of the boundary between the optimality region for *FLF* and the other regions remains. In particular, for moderate δ_b, as α_b increases, the firm can switch from *FLF* to *CE*, and vice versa. Thus, even under dynamic pricing, it is important for firms to understand how a prerelease advertising campaign promoting premium features impacts the choice of the business model. In Figure 8 we show one example to illustrate the above points. Given that insights from the constant pricing setting continue to hold under dynamic pricing, for brevity, we omit replicating additional discussions and figures.

Regardless of the magnitude of WOM effects, as long as w is finite, there will be a region in the parameter space where *S* strongly dominates the other strategies (when α and $\alpha_b + \delta_b$ are very low). However, different from the constant pricing scenario, in the limit, under extreme WOM effects (perfect learning—$w = \infty$), *S* and *CE* will converge. Under both strategies, when priors are low, a negligible installed base in period 1 is sufficient to induce perfect learning. Nevertheless, unlike in the case of constant pricing, under *CE* the firm can now adjust the price in period 2 and take full advantage of updated consumer valuations. As discussed in §3.2.2, there can be many potential factors that affect the strength of WOM effects.

Last, whereas in this section we consider dynamic pricing with commitment (where customers take period 2 price into account when making their purchase decisions in period 1), in online supplement D, we solve also for the optimal strategy under each business model when there is no price commitment and $w = 1$. In the latter scenario, given the bounded rationality assumptions, customers cannot anticipate period 2 pricing at the beginning of period 1 (because they do not have information about consumer type distribution and do not have resources to keep track of all the potential ways in which the market can unfold). As such, under the no-commitment scenario, we consider myopic customers. We point out that the optimal pricing under *CE* in region *R*1 under price commitment (part (a) of Proposition 6) is identical to the optimal price under no commitment (Proposition D1). That is because in region *R*1, in spite of price commitment, it is optimal to price such that IC constraint is not binding in period 1 and IR constraints are binding in periods 1 and 2 for the marginal customers. Furthermore, the region where *S* dominates *CE*, and the optimal seeding ratio and pricing in this region, are identical across the two commitment scenarios (as can be seen from Propositions 7 and D2). In the scenario of no commitment, the key insights that we derived throughout the paper continue to apply. As such, the derivations of optimal strategies for this scenario have been moved to online supplement D and the additional discussions have been omitted for brevity.

6. Conclusions

Motivated by practical examples in the software industry, we investigate formally the economics of *free* offers when *both* free and non-free offers come under perpetual licensing in scenarios where customers do not know initially their true valuation for software functionality. In particular, we focus on two popular business models that involve a free component—feature-limited freemium (*FLF*) and uniform seeding (*S*)—and benchmark their performance against a conventional business model that sells all the functionality bundled inside one product without any free offer (*CE*). The three strategies are formalized under a novel, unifying, multiperiod consumer valuation learning framework that accounts for both WOM effects and experience-based learning in the context of boundedly rational consumers. To the best of our knowledge, this is the first paper that compares the three business models. Our study does not only reveal that offering some form of free consumption may dominate *CE*, but also sets out to identify *which* of the models with an associated free component are best depending on the market context. In that sense, this paper makes an important contribution toward helping firms in their business model search.

As mentioned in §1, business model search is intricately related to development and advertising efforts, and our analysis provides important and nontrivial managerial insights as to how the level of premium functionality (b), consumer priors on their valuation for functionality in the two modules (α_a and α_b), cross-module synergies (δ_b), and strength of WOM effects (w) *jointly* impact firm's subsequent choices. Under both constant and dynamic pricing, for moderate strength of WOM signals, we derive the equilibria for each model (*CE*, *FLF*, *S*), which, even taken in isolation, offer important guidelines for any firm choosing to implement one of these models. Furthermore, under constant pricing, we fully characterize conditions under which each model is best and later discuss how the trade-offs that determine optimality regions persist under dynamic pricing. We find that every model has its own region of the feasible parameter space where it is optimal. In particular, *S* will be optimal when consumers significantly underestimate the value of functionality and cross-module synergies are weak. When either cross-module synergies are stronger or initial priors are higher, the firm will decide between *CE* and *FLF*. Furthermore, we identify nontrivial switching dynamics from one optimality region to another depending in complex ways on the initial consumer beliefs about the value of the embedded functionality. For example, when the strength of WOM effects is not extremely high, in regions where customers start with low prior valuations and cross-module synergies are not very high, ceteris paribus, increasing consumer priors on the value of *premium* functionality induces the firm to progressively switch from *S* to *FLF* to *CE* and, ultimately, back to *FLF* (even more complex switches are discussed when we consider alternative parameterizations of δ_b in §4.5). Thus, there are regions of the parameter space where, all other things equal, *FLF* is optimal when the prior on premium functionality is either relatively low or high, but not in between. On the other hand, if customers' initial valuations for the bundle are already high, then increases in the initial valuation of the premium functionality will not induce the firm to change strategy. Under constant pricing, these switches are also mirrored by transitions back and forth between taking advantage of WOM effects (inducing period 2 sales) and selling solely in period 1. We also explore the sensitivity of our results with respect to various parameterizations of cross-module synergies, strength of WOM effects, number of periods, and dynamic pricing. We find that stronger WOM effects or more periods lead to an expansion of the parameter space under which seeding is optimal in parallel with a decrease in the optimal seeding ratio. Moreover, under dynamic pricing, in the case of *CE* (and implicitly *FLF*), we uncover a nontrivial behavior of the price in the second period—it is decreasing in initial consumer valuation beliefs when WOM effects are strong and the prior is low (but not too low). However, this is not the case under weak WOM effects. In addition, our analysis reveals that, under *CE*, lower initial valuation scenarios should be approached using a penetration pricing strategy, whereas higher initial valuation scenarios are better tackled under a price skimming strategy. The region where penetration pricing is preferred expands as WOM effects grow stronger.

Since *FLF* can be seen as a particular case of versioning (where the basic quality version is offered for free), our results also contribute toward understanding the complex versioning problem for digital goods in the context of WOM effects, cross-module synergies, and multiperiod product life. Our paper takes as given the functionality level and how it is split between the two modules, and presents conditions when *CE* is dominated by *FLF*. Traditional versioning studies, while considering simpler settings (in general without WOM effects or multiple-period scenarios), often times endogenize the development of the quality and/or how much quality each version captures. Extending our analysis in this direction is beyond the scope of this paper but would be an interesting exercise for future research.

Our analysis can be further extended in multiple ways. For example, one could consider competition when a start-up embraces freemium to compete with an incumbent. In addition, freemium models associated with other licensing policies may also be explored. Throughout this study, we focused on software products characterized by a one-time purchase, where users run the software on their own machines and the developer does not invest additional resources to support the consumption of the product. For simplicity we do not consider quality improvement and maintenance via patching. However, freemium applicability extends beyond such products, in particular branching into the rapidly growing markets for software-as-a-service products, where adoption dynamics are slightly different and the revenue model is in many cases subscription based, involving recurring payments from the installed base. In such markets, service providers may incur nonnegligible operational costs associated with running the application (on own hardware infrastructure or on stable environments sourced from platform-as-a-service providers such as Salesforce.com or infrastructure-as-a-service providers such as Amazon or Rackspace). Offering a feature-limited freemium model involves bleeding costs from supporting the service for a mass of nonpaying customers for the entire product lifecycle, posing challenges to the viability of the model. Sometimes, service providers cover a portion of these bleeding costs by offering an ad-supported model. It would be very interesting to explore conditions under which freemium models would dominate *CE* models in the software-as-a-service markets. Last, our bounded rationality assumptions can be further relaxed. Future studies could explore what happens under different (e.g., Bayesian) valuation learning frameworks.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/isre.2013.0508.

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References

Anderson C (2009) *Free: The Future of a Radical Price* (Hyperion, New York).

App Promo (2012) The necessity of mobile app marketing: What it really takes to succed with a mobile app. http://app-promo.com/wp-content/uploads/2012/04/AppPromo-TheNecessityofMobileAppMarketing.pdf.

August T, Niculescu MF, Shin H (2013) Cloud implications on software network structure and security risks. Working paper, U. C. San Diego and Georgia Institute of Technology (Version Sep. 6, 2013). SSRN: http://ssrn.com/abstract=1933618.

Bawa K, Shoemaker R (2004) The effects of free sample promotions on incremental brand sales. *Marketing Sci.* 23(3):345–363.

Bhargava H, Choudhary V (2001) Information goods and vertical differentiation. *J. Management Inform. Systems* 18(2):89–106.

Bhargava H, Choudhary V (2008) When is versioning optimal for information goods? *Management Sci.* 54(5):1029–1035.

Biyalogorsky E, Boulding W, Staelin R (2006) Stuck in the past: Why managers persist with new product failures. *J. Marketing* 70(2):108–121.

Chaiken S (1980) Heuristic versus systematic information processing and the use of source versus message cues in persuasion. *J. Personality Soc. Psych.* 39(5):752–766.

Chaiken S, Maheswaran D (1994) Heuristic processing can bias systamatic processing: Effects of source credibility, argument ambiguity, and task importance on attitude judgement. *J. Personality Soc. Psych.* 66(3):460–473.

Chatterjee R, Eliashberg J (1990) The innovation diffusion process in a heterogeneous population: A micromodeling approach. *Management Sci.* 36(9):1057–1079.

Chellappa R, Shivendu S (2005) Managing piracy: Pricing and sampling strategies for digital experience goods in vertically segmented markets. *Inform. Systems Res.* 16(4):400–417.

Chellappa RK, Jia J (2013) Versioning and competition. Working paper, Emory University and Hong Kong University of Science and Technology (Version: Jun. 19, 2013).

Chellappa RK, Mehra A (2013) Versioning 2.0: A product line and pricing model for information goods under usage constraints and with R&D costs. *Conf. Inform. Systems Technol., Minneapolis, MN*, Oct. 5–6, http://innovisops.com/papers/cist2013_submission_107.pdf.

Chen Y, Xie J (2008) Online consumer review: Word-of-mouth as a new element of marketing communication mix. *Management Sci.* 54(3):477–491.

Chen YJ, Seshadri S (2007) Product development and pricing strategy for information goods under heterogeneous outside opportunities. *Inform. Systems Res.* 18(2):150–172.

Chen YX, Iyer G, Pazgal A (2010) Limited memory, categorization, and competition. *Marketing Sci.* 29(4):650–670.

Cheng HK, Liu Y (2012) Optimal software free trial strategy: The impact of network externalities and consumer uncertainty. *Inform. Systems Res.* 23(2):488–504.

Cheng HK, Tang QC (2010) Free trial or no free trial: Optimal software product design with network effects. *Eur. J. Oper. Res.* 205(2):437–447.

Chevalier JA, Mayzlin D (2006) The effect of word of mouth on sales: Online book reviews. *J. Marketing Res.* 43(3):345–354.

Choudhary V (2007) Comparison of software quality under perpetual licensing and software as a service. *J. Management Inform. Systems* 24(2):141–165.

Claburn T (2013) Apple OS X Mavericks sees fast adoption. *InformationWeek* (Oct. 24). http://www.informationweek.com/desktop/apple-os-x-mavericks-sees-fast-adoption/d/d-id/1112065?.

Conner KR (1995) Obtaining strategic advantage from being imitated: When can encouraging clones pay? *Management Sci.* 41(2):209–225.

Crookes D (2012) Apple makes try-before-you-buy possible for app users. The Independent. March 8. http://www.independent.co.uk/life-style/gadgets-and-tech/news/apple-makes-trybeforeyoubuy-possible-for-app-users-7545826.html.

Datamonitor (2011) Software: Global industry guide. Dec. 1.

Dellarocas C (2003) The digitization of word of mouth: Promise and challenges of online feedback mechanisms. *Management Sci.* 49(10):1407–1424.

Dellarocas C, Zhang X, Awad N (2007) Exploring the value of online product reviews in forecasting sales: The case of motion pictures. *J. Interactive Marketing* 21(4):23–45.

Dey D, Lahiri A (2013) The DLC strategy: Go vertical in a horizontal market. Working paper, Univ. of Washington (Version Nov. 5). SSRN: http://ssrn.com/abstract=2066194.

Dey D, Lahiri A, Liu D (2013) Consumer learning and time-locked trials of software products. *J. Management Inform. Systems*, 30(2):239–267.

Distimo, MEF (2013) Distimo and MEF—Global trends in established and growth markets. September. http://www.distimo.com/publications.

Dodson JA, Muller E (1978) Models of new product diffusion through advertising and word-of-mouth. *Management Sci.* 24(15):1568–1578.

Dou Y, Niculescu MF, Wu DJ (2013) Engineering optimal network effects via social media features and seeding in markets for digital goods and services. *Inform. Systems Res.* 24(1):164–185.

Duan W, Gu B, Whinston AB (2008) Do online reviews matter?—An empirical investigation of panel data. *Decision Support Systems* 45(4):1007–1016.

Ellison G, Fudenberg D (1995) Word-of-mouth communication and social learning. *Quar. J. Econom.* 110(1):93–125.

Forman C, Ghose A, Wiesenfeld B (2008) Examining the relationship between reviews and sales: The role of reviewer identity disclosure in electronic markets. *Inform. Systems Res.* 19(3):291–313.

Galeotti A, Goyal S (2009) Influencing the influencers: A theory of strategic diffusion. *RAND J. Econom.* 40(3):509–532.

Ghose A, Ipeirotis PG (2011) Estimating the helpfulness and economics impact of product reviews: Mining text and reviewer characteristics. *IEEE Trans. Knowledge and Data Engrg.* 23(10):1498–1512.

Ghose A, Sundararajan A (2005) Software versioning and quality degradation? An exploratory study of the evidence. Working paper, New York Univeristy (Version Aug. 2005). SSRN: http://ssrn.com/abstract=786005.

Heiman A, Muller E (1996) Using demonstration to increase new product acceptance: Controlling demonstration time. *J. Marketing Res.* 33(4):422–430.

Heiman A, McWilliams B, Shen Z, Zilberman D (2001) Learning and forgetting: Modeling optimal product sampling over time. *Management Sci.* 47(4):532–546.

Ingraham N (2013) Apple announces 1 million apps in the App Store, more than 1 billion songs played on iTunes radio. The Verge, Oct. 22. http://www.theverge.com/2013/10/22/4866302/apple-announces-1-million-apps-in-the-app-store.

Internet Systems Consortium (2013) Internet domain survey. July. http://www.isc.org/services/survey/.

Internet World Stats (2013) World Internet usage and population statistics. http://www.internetworldstats.com/stats.htm.

Jain D, Mahajan V, Muller E (1995) An approach for determining optimal product sampling for the diffusion of a new product. *J. Prod. Innovation Management* 12:124–1356.

Jiang Z, Sarkar S (2010) Speed matters: The role of free software offer in software diffusion. *J. Management Inform. Systems* 26(3):207–239.

Jing B (2011) Social learning and dynamic pricing of durable goods. *Marketing Sci.* 30(5):851–865.

Jonas E, Schulz-Hardt S, Frey D, Thelen N (2001) Confirmation bias in sequential information search after preliminary decisions: An expansion of dissonance theoretical research on selective exposure to information. *J. Personality Soc. Psych.* 80(4):557–571.

Jones R, Mendelson H (2011) Information goods vs. industrial goods: Cost structure and competition. *Management Sci.* 57(1):164–176.

Kahneman D (2003) Maps of bounded rationality: Psychology of behavioral economics. *Amer. Econom. Rev.* 93(5):1449–1475.

Kalish S (1983) Monopolist pricing with dynamic demand and production cost. *Marketing Sci.* 2(2):135–159.

Kalish S (1985) A new product adoption model with price, advertising, and uncertainty. *Management Sci.* 31(12):1569–1585.

Kang YS, Herr PM (2006) Beauty and the beholder: Toward an integrative model of communication source effects. *J. Consumer Res.* 33(1):123–130.

Kelman HC (1961) Processes of attitute change. *Public Opinion Quart.* 25:57–78.

Lehmann DR, Esteban-Bravo M (2006) When giving some away makes sense to jump-start the diffusion process. *Marketing Lett.* 17(4):243–254.

Libai B, Muller E, Peres R (2005) The role of seeding in multi-market entry. *Int. J. Res. Marketing* 22(4):375–393.

Libai B, Muller E, Peres R (2013) Decomposing the value of word-of-mouth seeding programs: Acceleration versus expansion. *J. Marketing Res.* 50(2):161–176.

Mackie D, Worth L, Asuncion A (1990) Proccessing of persuasive in-group messages. *J. Personality Soc. Psych.* 58(5):812–822.

Mahajan V, Muller E, Kerin RA (1984) Introduction strategy for new products with positive and negative word-of-mouth. *Management Sci.* 30(12):1389–1404.

Mankiw NG, Reis R (2002) Sticky information versus sticky prices: A proposal to replace the new Keynesian Phillips curve. *Quart. J. Econom.* 117(4):1295–1328.

Menon T, Blount S (2003) The messenger bias: A relational model of knowledge valuation. *Res. Organ. Behav.* 25:137–186.

Miller CC (2009) Ad revenue on the Web? No sure bet. *New York Times*. http://www.nytimes.com/2009/05/25/technology/start-ups/25startup.html?_r=1.

Morris S, Shin HS (2006) Inertia of forward-looking expectations. *Amer. Econom. Rev.* 96(2):152–157.

Mussa M, Rosen S (1978) Monopoly and product quality. *J. Econom. Theory* 18:301–317.

Niculescu MF, Shin H, Whang S (2012) Underlying consumer heterogeneity in markets for subscription-based IT services with network effects. *Inform. Systems Res.* 23(4):1322–1341.

Osborne MJ, Rubinstein A (1998) Games with procedurally rational players. *Amer. Econom. Rev.* 88(4):834–847.

Petty RE, Wegener DT, White PH (1998) Flexible correction processes in social judgement: Implications for persuasion. *Soc. Cognition* 16(1):93–113.

Pornpitakpan C (2004) The persuasiveness of source credibility: A critical review of five decades' evidence. *J. Appl. Soc. Psych.* 34(2):243–281.

Rabin M, Schrag JL (1999) First impressions matter: A model of confirmatory bias. *Quart. J. Econom.* 114(1):37–82.

Radner R, Radunskaya A, Sundararajan A (2013) Dynamic pricing of network goods with boundedly rational consumers. *Proc. Natl. Acad. Sci. USA*, ePub ahead of print December 23, http://www.pnas.org/content/early/2013/12/18/1319543110.

Raghunathan S (2000) Software editions: An application of segmentation theory to the packaged software market. *J. Management Inform. Systems* 17(1):87–114.

Riggins FJ (2003) Market segmentation and information development costs in a two-tiered fee-based and sponsorship-based web site. *J. Management Inform. Systems* 19(3):69–86.

Rubinstein A (1993) On price recognition and computational complexity in a monopolistic model. *J. Political Econom.* 101(3):473–484.

Rubinstein A (1997) *Modeling Bounded Rationality* (MIT Press, Cambridge, MA).

Shivendu S, Zhang Z (2011) Versioning strategy of information goods with network externality in the presence of piracy. *Proc. 45th Annual Hawaii Internat. Conf. System Sci.* (IEEE Computer Society, Washington, DC), 4572–4578.

Shivendu S, Zhang Z (2012) Vertically differentiated markets with costly under-provisioning: The case of versioning in software industry. Working paper, U. C. Irvine (Version Dec. 1, 2012). SSRN: http://ssrn.com/abstract=2236323.

Simon HA (1972) Theories of bounded rationality. *Decision Organ.* 1:161–176.

Spiegler R (2006) Competition over agents with boundedly rational expectations. *Theoret. Econom.* 1:207–231.

Trusov M, Bucklin RE, Pauwels K (2009) Effects of word-of-mouth versus traditional marketing: Findings from an Internet social networking site. *J. Marketing* 73(5):90–102.

Wei X, Nault B (2005) Product differentiation and market segmentation of information goods. Working paper, Univeristy of Calgary (Version Dec. 2, 2005). SSRN: http://ssrn.com/abstract=909604.

Wei X, Nault B (2013) Experience information goods: "Version-to-upgrade." *Decision Support Systems* 56:494–501.

Whittaker Z (2013) *WWDC'13: Apple Keynote, by the Numbers* (ZDNet), Jun. 10. http://www.zdnet.com/wwdc-13-apple-keynote-by-the-numbers-7000016583.

Wu DJ, Ding M, Hitt LM (2013) IT implementation contract design: Analytical and experimental investigation of IT value, learning, and contract structure. *Inform. Systems Res.* 24(3): 787–801.

Wu S, Chen P (2008) Versioning and piracy control for digital information goods. *Oper. Res.* 56(1):157–172.

Zhang J, Seidmann A (2010) Perpetual versus subscription licensing under quality uncertainty and network externality effects. *J. Management Inform. Systems* 27(1):39–68.

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"Economics of Free Under Perpetual Licensing: Implications for the Software Industry"

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Proofs of Main and Supporting Results. Additional Discussions.

A Summary of Key Notation

Symbol	Explanation
CE, FLF, S	Charge-for-everything, feature-limited freemium, uniform seeding.
θ	Customer type.
a, b, c	Per-period real benefit factors from modules A, B, and bundle $A\&B$. Heterogeneous per-period consumer benefits are obtained by multiplying these factors with the consumer type.
a_0, a_1, a_2, b_0, b_1, b_2, c_0, c_1, c_2	Consumer beliefs about the real value of parameters a, b, c at various points in time.
α_a, α_b, α	Factors by which customers deviate in their initial beliefs from the true values of parameters a, b, c.
N_1, N_2	Installed base of *paying* customers at the end of various periods.
k	Seeding ratio under uniform seeding.
$\Delta(b)$, $b\delta_b$	General and specific forms for experience-based learning under FLF
p, p_1, p_2	Price under constant pricing or period-specific prices under dynamic pricing.
π_{CE}, π_{FLF}, π_S	Profits under various models.
w	Strength of WOM effects - captures ability of WOM to influence attitude and behavior of consumers.
T	Number of time periods.

B Constant Pricing with Commitment

Proof of Lemma 1. Follows immediately from the equilibrium discussion in §4.1. □

Proof of Lemma 2. Based on Lemma 1, we know that $\theta_2 < \theta_1$ if and only if $c_2 > 2c_1$. Given that, under the considered bounded rationality assumptions specified in §3.3, customers do not anticipate a change in their valuation for the product, under constant pricing the firm is constrained to price below the valuation of the highest type believes the product carries over two periods of usage, i.e. $2c\alpha$. Thus, $N_1 = 1 - \theta_1 = 1 - \frac{p}{2c\alpha}$, we have $c_2 = c\alpha + \left(1 - \frac{p}{2c\alpha}\right)(c - c\alpha) = c - \frac{p(1-\alpha)}{2\alpha}$. Therefore, by simple manipulation, we obtain:

$$c_2 > 2c_1 \;\Leftrightarrow\; c(1-2\alpha) > \frac{p(1-\alpha)}{2\alpha}. \tag{B.1}$$

Now we move ahead to prove the result in Lemma 2.

Direction '$\Rightarrow$'. Suppose adoption extends to period 2. Then, obviously, feasibility constraint $p < 2c\alpha$ must be satisfied. We have $c_2 > 2c_1$. Note that, as discussed in §4.1, when $\alpha \geq 1$ consumers never adopt in period 2 (they either adopt in period 1 and own the product in period 2 or do not adopt at all). Thus, if they adopt in period 2, it must be the case that $\alpha < 1$. From equation (B.1), it immediately follows that $\alpha < \frac{1}{2}$. Rewriting equation (B.1) immediately yields $p < \frac{2\alpha c(1-2\alpha)}{1-\alpha} < 2c\alpha$.

Direction '$\Leftarrow$'. Result follows immediately from the conditions. First, since $0 < \frac{1-2\alpha}{1-\alpha} < 1$, implicitly the price feasibility constraint $p < 2c\alpha$ is satisfied. Next, it is easy to see that (B.1) is satisfied, i.e., $c_2 > 2c_1$. Then, we immediately have $\theta_2 < \theta_1$. □

Proof of Proposition 1. We consider multiple cases:

Case 1: $\frac{1}{2} \leq \alpha$. In this case, as per Lemma 2, there is no additional adoption in period 2. For that reason:

$$\pi_{CE} = pN_1 = p\left(1 - \frac{p}{2c_1}\right), \tag{B.2}$$

where $c_1 = c\alpha$. Solving, we obtain $p^*_{CE} = c\alpha$ and $\pi_{CE}(p^*) = \frac{c\alpha}{2}$. We note that condition $p < 2c\alpha$ is satisfied, making this a feasible solution.

Case 2: $\alpha < \frac{1}{2}$. Here we consider two subcases:

a. $p \geq \frac{2c\alpha(1-2\alpha)}{1-\alpha}$. In this case, as per Lemma 2, there is no adoption in period 2. Thus, similar to Case 1, we have again $\pi_{CE} = pN_1 = p\left(1 - \frac{p}{2c_1}\right)$. Solving under constraints $\alpha < \frac{1}{2}$ and $p \geq \frac{2c\alpha(1-2\alpha)}{1-\alpha}$ we obtain:

$$p^*_{CE,a} = \begin{cases} c\alpha, & \text{if } \frac{1}{3} < \alpha < \frac{1}{2}, \\ \frac{2c\alpha(1-2\alpha)}{1-\alpha}, & \text{if } \alpha \leq \frac{1}{3}. \end{cases} \tag{B.3}$$

and

$$\pi_{CE}(p^*_{CE,a}) = \begin{cases} \frac{c\alpha}{2} & , \text{ if } \frac{1}{3} < \alpha < \frac{1}{2}, \\ \frac{2c\alpha^2(1-2\alpha)}{(1-\alpha)^2} & , \text{ if } \alpha \leq \frac{1}{3}. \end{cases} \tag{B.4}$$

b. $p \leq \frac{2c\alpha(1-2\alpha)}{1-\alpha}$. In this case, the problem gets more complex. According to Lemma 2, we have adoption in both periods when $p < \frac{2c\alpha(1-2\alpha)}{1-\alpha}$. We can also consider that adoption with mass 0 happens in period 2 when $p = \frac{2c\alpha(1-2\alpha)}{1-\alpha}$ since, in this case, $c_2 = 2c_1$ and $\theta_2 = \theta_1 = \frac{p}{c_2}$. Including the upper bound on the price interval allows us to derive the precise optimal price under constrained profit maximization. Potential consumers who have not adopted in period 1 expect value c_2 by updating c_1 at the end of period 1. We have $c_2 = c_1 + N_1(c - c_1) =$

$c_1 + \left(1 - \frac{p}{2c_1}\right)(c - c_1) = c - \frac{p(1-\alpha)}{2\alpha}$. Therefore, we can write the profit function as:

$$\pi_{CE} = pN_2 = p\left(1 - \frac{p}{c_2}\right) = p\left(1 - \frac{p}{c - \frac{p(1-\alpha)}{2\alpha}}\right). \quad \text{(B.5)}$$

The solutions p_1 and p_2 to the equation $\frac{\partial \pi}{\partial p}(p) = 0$ are:

$$p_{1,2} \quad = \quad \frac{2c\alpha}{1-\alpha}\left(1 \pm \sqrt{\frac{2\alpha}{1+\alpha}}\right) > 0, \quad \text{(B.6)}$$

and we have:

$$\frac{\partial \pi_{CE}}{\partial p} \begin{cases} > 0 & , \text{ if } p < p_1, \\ = 0 & , \text{ if } p = p_1, \\ < 0 & , \text{ if } p_1 < p < p_2, \\ = 0 & , \text{ if } p = p_2, \\ > 0 & , \text{ if } p > p_2. \end{cases}$$

Therefore, $\pi_{CE}(p)$ is increasing over $[0, p_1]$, decreasing over $[p_1, p_2]$ and increasing over $[p_2, \infty)$. However, it is straightforward to prove that:

$$0 < \frac{2c\alpha(1-2\alpha)}{1-\alpha} < 2\alpha c < p_2. \quad \text{(B.7)}$$

Thus, on the interval $\left[0, \frac{2c\alpha(1-2\alpha)}{1-\alpha}\right]$, $\pi_{CE}(p)$ is either *increasing* or *unimodal* with peak p_1. Since we are already imposing constraint $\alpha < \frac{1}{2}$, we have:

$$\frac{2c\alpha(1-2\alpha)}{1-\alpha} < p_1 \Leftrightarrow \frac{1}{\sqrt{3}+1} < \alpha < \frac{1}{2}.$$

Therefore, imposing constraints $\alpha < \frac{1}{2}$ and $p \leq \frac{2c\alpha(1-2\alpha)}{1-\alpha}$, we obtain the following solutions:

$$p^*_{CE,b} = \begin{cases} \frac{2c\alpha(1-2\alpha)}{1-\alpha} & , \ \frac{1}{\sqrt{3}+1} < \alpha < \frac{1}{2}, \\ p_1 = \frac{2c\alpha}{1-\alpha}\left(1 - \sqrt{\frac{2\alpha}{1+\alpha}}\right) & , \ \alpha \leq \frac{1}{\sqrt{3}+1}, \end{cases} \quad \text{(B.8)}$$

and

$$\pi_{CE}(p^*_{CE,b}) = \begin{cases} \frac{2c\alpha^2(1-2\alpha)}{(1-\alpha)^2} & , \ \frac{1}{\sqrt{3}+1} < \alpha < \frac{1}{2}, \\ \frac{2c\alpha(\sqrt{1+\alpha}-\sqrt{2\alpha})^2}{(1-\alpha)^2} & , \ \alpha \leq \frac{1}{\sqrt{3}+1}. \end{cases} \quad \text{(B.9)}$$

Reconciling Cases 2.a and 2.b. We have finished analyzing Cases 2.a and 2.b and we now need to further reconcile them, i.e., characterize the *choice* of the firm whether to allow consumers to adopt in the second period or not.

(i) $\alpha \leq \frac{1}{3}$. Using the fact that, for any $x \neq y$, we have $x^2 + y^2 > 2xy$, we immediately obtain

	Firm		Consumer Paid Adoption Pattern
	p^*_{CE}	$\pi_{CE}(p^*_{CE})$	
$\frac{1}{2} \leq \alpha$	$c\alpha$	$\frac{c\alpha}{2}$	Per. 1
$\frac{1}{\sqrt{3}+1} < \alpha < \frac{1}{2}$	$c\alpha$	$\frac{c\alpha}{2}$	Per. 1
$13 - 4\sqrt{10} \leq \alpha \leq \frac{1}{\sqrt{3}+1}$	$c\alpha$	$\frac{c\alpha}{2}$	Per. 1
$\frac{1}{3} < \alpha < 13 - 4\sqrt{10}$	$\frac{2c\alpha}{1-\alpha}\left(1 - \sqrt{\frac{2\alpha}{1+\alpha}}\right)$	$\frac{2c\alpha(\sqrt{1+\alpha}-\sqrt{2\alpha})^2}{(1-\alpha)^2}$	Per. 1,2
$0 < \alpha \leq \frac{1}{3}$	$\frac{2c\alpha}{1-\alpha}\left(1 - \sqrt{\frac{2\alpha}{1+\alpha}}\right)$	$\frac{2c\alpha(\sqrt{1+\alpha}-\sqrt{2\alpha})^2}{(1-\alpha)^2}$	Per. 1,2

Table B1: Optimal Price - CE Model

$1 + \left(2\alpha + 2\alpha^2\right) > 2\sqrt{2\alpha(1+\alpha)}$. It immediately follows that:

$$\pi_{CE}(p^*_{CE,a}) = \frac{2c\alpha^2(1-2\alpha)}{(1-\alpha)^2} < \frac{2c\alpha(\sqrt{1+\alpha} - \sqrt{2\alpha})^2}{(1-\alpha)^2} = \pi_{CE}(p^*_{CE,b}).$$

In this case, the firm finds it optimal to allow customers to adopt also in the second period by setting a low enough price.

(ii) $\frac{1}{3} < \alpha < 13 - 4\sqrt{10}$. Then, using $1 - \alpha = (\sqrt{1+\alpha} - \sqrt{2\alpha})(\sqrt{1+\alpha} + \sqrt{2\alpha})$, it can be shown that:

$$\pi_{CE}(p^*_{CE,a}) = \frac{c\alpha}{2} < \frac{2c\alpha(\sqrt{1+\alpha} - \sqrt{2\alpha})^2}{(1-\alpha)^2} = \pi_{CE}(p^*_{CE,b}).$$

In this case, the firm finds it optimal to allow customers to adopt also in the second period by setting a low enough price.

(iii) $\alpha = 13 - 4\sqrt{10}$. Then, it can be shown that:

$$\pi_{CE}(p^*_{CE,a}) = \frac{c\alpha}{2} = \frac{2c\alpha(\sqrt{1+\alpha} - \sqrt{2\alpha})^2}{(1-\alpha)^2} = \pi_{CE}(p^*_{CE,b}).$$

We are assuming that, all other things equal, the firm prefers to get the revenue faster (argument also mentioned in the main text immediately after Proposition 1 in footnote 5). Therefore, in this case, the firm will choose a high price such that there is no adoption in period 2.

(iv) $13 - 4\sqrt{10} < \alpha \leq \frac{1}{\sqrt{3}+1}$. It can be shown that:

$$\pi_{CE}(p^*_{CE,a}) = \frac{c\alpha}{2} > \frac{2c\alpha(\sqrt{1+\alpha} - \sqrt{2\alpha})^2}{(1-\alpha)^2} = \pi_{CE}(p^*_{CE,b}).$$

In this case the firm will price so that there is no adoption in period 2.

(v) $\frac{1}{\sqrt{3}+1} < \alpha < \frac{1}{2}$. Then it is easy to show that:

$$\pi_{CE}(p^*_{CE,a}) = \frac{c\alpha}{2} > \frac{2c\alpha^2(1-2\alpha)}{(1-\alpha)^2} = \pi_{CE}(p^*_{CE,b}).$$

In this case the firm will price so that there is no adoption in period 2.

Cases 1 and 2 are put together in Table B1, illustrating firm's optimal strategy. □

Lemma B1. *Under S, for any feasible price satisfying (7), adoption extends to the second period if and only if one of the following two cases occurs:*
(a) $\frac{k}{1+k} \leq \alpha < \frac{1}{2}$ *and* $p < \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$, *or* *(b)* $0 < \alpha < \frac{k}{1+k}$.

Proof. Direction '⇒'. Suppose adoption extends to period 2. Constraint (7) must be satisfied. Note that, similar to the CE case, when $\alpha \geq 1$ consumers never adopt in period 2 (they either adopt in period 1 and own the product in period 2 or do not adopt at all). This is because overestimation induces a downwards adjustment of the consumers' WTP. Thus, if they adopt in period 2, it must be the case that $\alpha < 1$. We consider two cases:

(a) $\frac{k}{1+k} \leq \alpha$. Then we have $2c\alpha \geq c[\alpha + (1-\alpha)k]$. Therefore, constraint (7) translates into $p < 2c\alpha$. Under this constraint, adoption will start in period 1 and we have $\theta_1 = \frac{p}{2c\alpha}$. Given that $N_1 + k = k + (1-k)(1-\theta_1) = k + (1-k)\left(1 - \frac{p}{2c\alpha}\right) = 1 - \frac{p(1-k)}{2c\alpha}$, we immediately obtain $c_2 = c - \frac{p(1-k)(1-\alpha)}{2\alpha}$. In this case, we know that $\theta_2 < \theta_1$ if and only if $c_2 > 2c_1$.

$$c_2 > 2c_1 \;\Leftrightarrow\; c(1-2\alpha) > \frac{p(1-k)(1-\alpha)}{2\alpha}. \tag{B.10}$$

Given that $k \in [0, 1)$ and, as mentioned above, we only consider the case $\alpha < 1$, from (B.10) we have $c(1-2\alpha) > 0$, or $\alpha < \frac{1}{2}$. Next, by rewriting equation (B.10), we obtain $p < \frac{2\alpha c(1-2\alpha)}{(1-\alpha)(1-k)}$.

(b) $0 < \alpha < \frac{k}{1+k}$. Then $0 < 2c\alpha < c[\alpha+(1-\alpha)k]$, and constraint (7) translates into $p < c[\alpha+k(1-\alpha)]$. We have two subcases:

(i) $p < 2c\alpha$. In this case, adoption starts in period 1. Similar to case (a), adoption in period 2 yields constraints $\alpha < \frac{1}{2}$ and $p < \frac{2\alpha c(1-2\alpha)}{(1-\alpha)(1-k)}$. However, since $\frac{k}{1+k} < \frac{1}{2}$ and $\alpha < \frac{k}{1+k}$, we have $1 - 2\alpha \geq (1-\alpha)(1-k)$ and period 2 adoption constraints are automatically satisfied.

(ii) $2c\alpha \leq p < c[\alpha + k(1-\alpha)]$. In this case, there is no adoption in period 1 (negligible adoption of mass 0 happens when $p = 2c\alpha$); only in period 2. $c_2 = c[\alpha + k(1-\alpha)] \geq p$. Thus, $\theta_2 = \frac{p}{c[\alpha+k(1-\alpha)]} < 1 = \theta_1$. No additional constrains are necessary for adoption to occur in the second period.

Direction '⇐'. Follows immediately in each case from the conditions. □

Lemma B2. *Consider a fixed value $k \in (0, 1)$. Let us define the function:*

$$h_k(\alpha) \triangleq [\alpha + (1-\alpha)k]\left(\sqrt{2\alpha + (1-\alpha)(1-k)} + \sqrt{2\alpha}\right)^2 - 8\alpha.$$

Then, over the interval $\left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]$, *$h_k(\alpha)$ is strictly decreasing in α and equation $h_k(\alpha) = 0$ has a unique solution $\tilde{\alpha}(k)$.*

Proof. Differentiating $h_k(\alpha)$ we obtain:

$$\begin{aligned}\frac{\partial h_k}{\partial \alpha} &= (1-k)\left(\alpha(3+k)-k+1+2\sqrt{2\alpha}\sqrt{2\alpha+(1-\alpha)(1-k)}\right)\\ &\quad +(\alpha+k(1-\alpha))\left(3+k+\frac{4\alpha(1+k)+2(1-k)}{\sqrt{2\alpha}\sqrt{2\alpha+(1-\alpha)(1-k)}}\right)-8. \qquad \text{(B.11)}\end{aligned}$$

Consider $\alpha \in \left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]$, it can be shown that:

$$\alpha(3+k)-k+1+2\sqrt{2\alpha}\sqrt{2\alpha+(1-\alpha)(1-k)} < 3, \qquad \text{(B.12)}$$

$$(\alpha+k(1-\alpha))(3+k) \leq 4k, \qquad \text{(B.13)}$$

$$\frac{(\alpha+k(1-\alpha))(4\alpha(1+k)+2(1-k))}{\sqrt{2\alpha}\sqrt{2\alpha+(1-\alpha)(1-k)}} < 4. \qquad \text{(B.14)}$$

Putting together (B.11), (B.12), (B.13), and (B.14), we obtain:

$$\frac{\partial h_k}{\partial \alpha} < 3(1-k)+4k+4-8 = k-1 < 0,\ \forall \alpha \in \left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]. \qquad \text{(B.15)}$$

We show in the proof of Lemma B3 (to follow) that $h_k(\frac{k}{3+k}) \leq 0$ and $h_k\left(\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}\right) \geq 0$. Therefore equation $h_k(\alpha)=0$ has a unique solution $\tilde{\alpha}(k)$ in this interval. □

Lemma B3. *Under S model, for a given seeding ratio $k \in [0,1)$, the firm's optimal pricing strategy and the associated profit are:*

	$0 < \alpha < \tilde{\alpha}(k)$	$\tilde{\alpha}(k) \leq \alpha < \frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2}$	$\frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2} \leq \alpha$
$p_S^*(k)$	$\frac{c[\alpha+(1-\alpha)k]}{2}$	$\frac{2c\alpha}{(1-\alpha)(1-k)}\left(1-\sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right)$	$c\alpha$
$\pi_S^*(k)$	$\frac{(1-k)c[\alpha+(1-\alpha)k]}{4}$	$\frac{2c\alpha\left(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha}\right)^2}{(1-k)(1-\alpha)^2}$	$\frac{c\alpha(1-k)}{2}$
Paid adoption	*only in per. 2*	*in both per. 1 and 2*	*only in per. 1*

where $\tilde{\alpha}(k)$ represents the unique solution over the interval $\left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]$ to the equation $h_k(\alpha)=0$, with $h_k(\alpha) = [\alpha+(1-\alpha)k]\left(\sqrt{2\alpha+(1-\alpha)(1-k)}+\sqrt{2\alpha}\right)^2 - 8\alpha$.

Proof. We first consider $k \in (0,1)$ and explore several cases:

Case 1: $\frac{1}{2} \leq \alpha$. In this case, as per Lemma B1, there is no additional adoption in period 2. Thus, $\pi_S = p(N_1-k) = p(1-k)\left(1-\frac{p}{2c\alpha}\right)$. It follows that $p_S^* = c\alpha$ and $\pi_S(p_S^*) = \frac{c\alpha(1-k)}{2}$.

Case 2. $\frac{k}{1+k} \leq \alpha < \frac{1}{2}$. We consider two subcases:

a. $p \geq \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$. In this case, as per Lemma B1, there is no adoption in period 2 and the profit functions is $\pi_S = p(N_1-k) = p(1-k)\left(1-\frac{p}{2c\alpha}\right)$. Solving under constraints $\alpha < \frac{1}{2}$ and $p \geq \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$ we obtain:

$$p^*_{S,a} = \begin{cases} c\alpha, & \text{if } \frac{1+k}{3+k} < \alpha < \frac{1}{2}, \\ \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}, & \text{if } \frac{k}{1+k} \leq \alpha \leq \frac{1+k}{3+k}. \end{cases} \tag{B.16}$$

$$\pi_S(p^*_{S,a}) = \begin{cases} \frac{c\alpha(1-k)}{2} & , \text{ if } \frac{1+k}{3+k} < \alpha < \frac{1}{2}, \\ \frac{2c\alpha(1-2\alpha)(\alpha+\alpha k-k)}{(1-k)(1-\alpha)^2} & , \text{ if } \frac{k}{1+k} \leq \alpha \leq \frac{1+k}{3+k}. \end{cases} \tag{B.17}$$

b. $p \leq \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$. When $p < \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$, according to Lemma B1, we have adoption in both periods. When $p = \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$, we can consider that adoption with mass 0 occurs in period 2. Profit is given by:

$$\pi = p(N_2 - k) = p(1-k)\left(1 - \frac{p}{c_2}\right) = p(1-k)\left(1 - \frac{p}{c - \frac{p(1-\alpha)(1-k)}{2\alpha}}\right). \tag{B.18}$$

The solutions $p_1 < p_2$ to the equation $\frac{\partial \pi}{\partial p}(p) = 0$ are:

$$p_{1,2} = \frac{2c\alpha}{(1-\alpha)(1-k)}\left(1 \pm \sqrt{\frac{2\alpha}{2\alpha + (1-\alpha)(1-k)}}\right) > 0, \tag{B.19}$$

and we have:

$$\frac{\partial \pi_S}{\partial p} \begin{cases} > 0 & , \text{ if } p < p_1, \\ = 0 & , \text{ if } p = p_1, \\ < 0 & , \text{ if } p_1 < p < p_2, \\ = 0 & , \text{ if } p = p_2, \\ > 0 & , \text{ if } p > p_2. \end{cases}$$

Therefore, $\pi_S(p)$ is increasing over $[0, p_1]$, decreasing over $[p_1, p_2]$ and increasing over $[p_2, \infty)$. It is straightforward to see that p_2 is infeasible under constraint (7) because $2c\alpha = \max\{2c\alpha, c[\alpha + k(1-\alpha)]\} < p_2$. On the interval $\left[0, \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}\right]$, $\pi_S(p)$ is either *increasing* or *unimodal* with peak p_1. Solving under constraints $\frac{k}{1+k} \leq \alpha < \frac{1}{2}$ and $p \leq \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$, we obtain the following solution:

$$p^*_{S,b} = \begin{cases} \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)} & , \frac{1}{\sqrt{3+k^2}+1-k} < \alpha < \frac{1}{2}, \\ p_1 = \frac{2c\alpha}{(1-\alpha)(1-k)}\left(1 - \sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right) & , \frac{k}{1+k} \leq \alpha \leq \frac{1}{\sqrt{3+k^2}+1-k}, \end{cases} \tag{B.20}$$

$$\pi_S(p^*_{S,b}) = \begin{cases} \frac{2c\alpha(1-2\alpha)(\alpha+\alpha k-k)}{(1-k)(1-\alpha)^2} & , \frac{1}{\sqrt{3+k^2}+1-k} < \alpha < \frac{1}{2}, \\ \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} & , \frac{k}{1+k} \leq \alpha \leq \frac{1}{\sqrt{3+k^2}+1-k}. \end{cases} \tag{B.21}$$

The formula for the second case ($\frac{k}{1+k} \leq \alpha \leq \frac{1}{\sqrt{3+k^2}+1-k}$) is derived using the fact that $(1-\alpha)(1-k) = (\sqrt{2\alpha + (1-\alpha(1-k)} - \sqrt{2\alpha})(\sqrt{2\alpha + (1-\alpha(1-k)} + \sqrt{2\alpha})$.

Reconciling subcases 2.a and 2.b. We reconcile the above subcases, characterizing the *choice* of the firm as whether to allow adoption in second period or not.

(i) $\frac{k}{1+k} \le \alpha \le \frac{1+k}{3+k}$. Since $k < 1$, it immediately follows that:

$$\pi_S(p^*_{S,a}) = \frac{2c\alpha(1-2\alpha)(\alpha+\alpha k-k)}{(1-k)(1-\alpha)^2} < \frac{2c\alpha(\sqrt{2\alpha+(1-\alpha)(1-k)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi_S(p^*_{S,b}).$$

(ii) $\frac{1+k}{3+k} < \alpha < \frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2}$. Then, it can be shown that:

$$\pi_S(p^*_{S,a}) = \frac{c\alpha(1-k)}{2} < \frac{2c\alpha(\sqrt{2\alpha+(1-\alpha)(1-k)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi_S(p^*_{S,b}).$$

(iii) $\alpha = \frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2}$. Then, it can be shown that:

$$\pi_S(p^*_{S,a}) = \frac{c\alpha(1-k)}{2} = \frac{2c\alpha(\sqrt{2\alpha+(1-\alpha)(1-k)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi_S(p^*_{S,b}).$$

Again, we assume the firm prefers earlier revenues to later revenues. Therefore, in this case, the firm will price so that there is no adoption in period 2.

(iv) $\frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2} < \alpha \le \frac{1}{\sqrt{3+k^2}+1-k}$. It can be shown that:

$$\pi_S(p^*_{S,a}) = \frac{c\alpha(1-k)}{2} > \frac{2c\alpha(\sqrt{2\alpha+(1-\alpha)(1-k)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi_S(p^*_{S,b}).$$

(v) $\frac{1}{\sqrt{3+k^2}+1-k} < \alpha < \frac{1}{2}$. Then, it can be shown that:

$$\pi_S(p^*_{S,a}) = \frac{c\alpha(1-k)}{2} > \frac{2c\alpha(1-2\alpha)(\alpha+\alpha k-k)}{(1-k)(1-\alpha)^2} = \pi_S(p^*_{S,b}).$$

Case 3. $0 < \alpha < \frac{k}{1+k}$. This case exists only when $k > 0$. Then, the feasible pricing constraint (7) becomes $0 < p < c[\alpha + (1-\alpha)k]$. We discuss two subcases:

a. $2c\alpha \le p < c[\alpha+(1-\alpha)k]$. In this case, adoption occurs only in period 2 and $\pi_S = p(N_2 - k) = p(1-k)\left(1 - \frac{p}{c[\alpha+(1-\alpha)k]}\right)$. Maximizing profit under constraint $2c\alpha \le p < c[\alpha+(1-\alpha)k]$, we obtain:

$$p^*_{S,c} = \begin{cases} 2c\alpha & \text{, when } \frac{k}{3+k} \le \alpha < \frac{k}{1+k}, \\ \frac{c[\alpha+(1-\alpha)k]}{2} & \text{, when } 0 < \alpha \le \frac{k}{3+k}, \end{cases} \tag{B.22}$$

$$\pi_S(p^*_{S,c}) = \begin{cases} (1-k)2c\alpha\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) & \text{, when } \frac{k}{3+k} \le \alpha < \frac{k}{1+k}, \\ \frac{(1-k)c[\alpha+(1-\alpha)k]}{4} & \text{, when } 0 < \alpha \le \frac{k}{3+k}. \end{cases} \tag{B.23}$$

b. $0 \le p \le 2c\alpha < c[\alpha+(1-\alpha)k]$. In this case adoption occurs in both periods. Similar to the analysis in case 2.b, we obtain the same solutions $p_1 < p_2$ described in (B.19) for the equation

$\frac{\partial \pi_S}{\partial p} = 0$. Again, p_2 is infeasibile. Then it can be shown that:

$$p^*_{S,d} = \begin{cases} \frac{2c\alpha}{(1-\alpha)(1-k)}\left(1 - \sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right) & , \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2} \leq \alpha < \frac{k}{1+k}, \\ 2c\alpha & , 0 < \alpha < \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \end{cases} \tag{B.24}$$

$$\pi_S(p^*_{S,d}) = \begin{cases} \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} & , \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2} \leq \alpha < \frac{k}{1+k}, \\ (1-k)2c\alpha\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) & , 0 < \alpha < \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}. \end{cases} \tag{B.25}$$

Reconciling subcases 3.a and 3.b.

First, it can be shown that $\frac{k}{3+k} > \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}$ when $k < 1$.

(i) $0 < \alpha < \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}$. Then:

$$\pi_S(p^*_{S,c}) = \frac{(1-k)c[\alpha+(1-\alpha)k]}{4} > 2c\alpha(1-k)\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) = \pi_S(p^*_{S,d}).$$

(ii) $\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2} \leq \alpha \leq \frac{k}{3+k}$. In this case, we know that:

$$\pi_S(p^*_{S,c}) = \frac{(1-k)c[\alpha+(1-\alpha)k]}{4} \quad \text{and} \quad \pi_S(p^*_{S,d}) = \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2}.$$

Then:

$$\pi_S(p^*_{S,c}) \geq \pi_S(p^*_{S,d}) \;\Leftrightarrow h_k(\alpha) \geq 0, \tag{B.26}$$

where function $h_k(\alpha)$ was defined in Lemma B2. From Lemma B2, we know that h_k is strictly decreasing and can change sign at most once on the interval $\left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]$. When $\alpha = \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}$, then $p^*_{S,d} = 2c\alpha$ and

$$\pi_S(p^*_{S,d}) = \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = (1-k)2c\alpha\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right).$$

It can be shown that:

$$\pi_S(p^*_{S,c}) = \frac{(1-k)c[\alpha+(1-\alpha)k]}{4} \geq 2c\alpha(1-k)\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) = \pi_S(p^*_{S,d}).$$

Therefore, $h_k\left(\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}\right) \geq 0$.

When $\alpha = \frac{k}{3+k}$, then $p^*_{S,c} = 2c\alpha$ and

$$\pi_S(p^*_{S,c}) = (1-k)2c\alpha\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) = \frac{(1-k)c[\alpha+(1-\alpha)k]}{4}.$$

	Firm		Consumer Paid Adoption Pattern
	$p_S^*(k)$	$\pi_S^*(k)$	
$\frac{1}{2} \leq \alpha$	$c\alpha$	$\frac{c\alpha(1-k)}{2}$	Per. 1
$\frac{1}{\sqrt{3+k^2}+1-k} < \alpha < \frac{1}{2}$	$c\alpha$	$\frac{c\alpha(1-k)}{2}$	Per. 1
$\frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2} \leq \alpha \leq \frac{1}{\sqrt{3+k^2}+1-k}$	$c\alpha$	$\frac{c\alpha(1-k)}{2}$	Per. 1
$\frac{1+k}{3+k} < \alpha < \frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2}$	$\frac{2c\alpha}{(1-\alpha)(1-k)}\left(1-\sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right)$	$\frac{2c\alpha(1-k)}{(\sqrt{2\alpha+(1-k)(1-\alpha)}+\sqrt{2\alpha})^2}$	Per. 1,2
$\frac{k}{1+k} \leq \alpha \leq \frac{1+k}{3+k}$	$\frac{2c\alpha}{(1-\alpha)(1-k)}\left(1-\sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right)$	$\frac{2c\alpha(1-k)}{(\sqrt{2\alpha+(1-k)(1-\alpha)}+\sqrt{2\alpha})^2}$	Per. 1,2
$\frac{k}{3+k} < \alpha < \frac{k}{1+k}$	$\frac{2c\alpha}{(1-\alpha)(1-k)}\left(1-\sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right)$	$\frac{2c\alpha(1-k)}{(\sqrt{2\alpha+(1-k)(1-\alpha)}+\sqrt{2\alpha})^2}$	Per. 1,2
$\tilde{\alpha}(k) \leq \alpha \leq \frac{k}{3+k}$	$\frac{2c\alpha}{(1-\alpha)(1-k)}\left(1-\sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right)$	$\frac{2c\alpha(1-k)}{(\sqrt{2\alpha+(1-k)(1-\alpha)}+\sqrt{2\alpha})^2}$	Per. 1, 2
$\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2} \leq \alpha < \tilde{\alpha}(k)$	$\frac{c[\alpha+(1-\alpha)k]}{2}$	$\frac{(1-k)c[\alpha+(1-\alpha)k]}{4}$	Per. 2
$0 < \alpha < \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}$	$\frac{c[\alpha+(1-\alpha)k]}{2}$	$\frac{(1-k)c[\alpha+(1-\alpha)k]}{4}$	Per. 2

Table B2: Optimal Price - S Model

It can be shown that:

$$\pi_S(p_{S,c}^*) = (1-k)2c\alpha\left(1-\frac{2\alpha}{\alpha+(1-\alpha)k}\right) \leq \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi_S(p_{S,d}^*).$$

Therefore, $h_k\left(\frac{k}{3+k}\right) \leq 0$. Since h_k is strictly decreasing over the interval $\left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]$, then the equation $h_k(\alpha) = 0$ has a unique solution $\tilde{\alpha}(k)$ in this interval. Consequently:

$$\begin{aligned} \pi_S(p_{S,c}^*) > \pi_S(p_{S,d}^*) \quad &\text{when} \quad \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2} \leq \alpha < \tilde{\alpha}(k), \\ \pi_S(p_{S,c}^*) = \pi_S(p_{S,d}^*) \quad &\text{when} \quad \alpha = \tilde{\alpha}(k), \\ \pi_S(p_{S,c}^*) < \pi_S(p_{S,d}^*) \quad &\text{when} \quad \tilde{\alpha}(k) < \alpha \leq \frac{k}{3+k}. \end{aligned}$$

When $\alpha = \tilde{\alpha}(k)$, similar as before, we assume the firm prefers earlier revenues more, and, as such, will price so that adoption happens in both period 1 and period 2.

(iii) $\frac{k}{3+k} < \alpha < \frac{k}{1+k}$. Then, it can be shown that:

$$\pi_S(p_{S,c}^*) = (1-k)2c\alpha\left(1-\frac{2\alpha}{\alpha+(1-\alpha)k}\right) < \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi_S(p_{S,d}^*).$$

Table B2 summarizes the results in Cases 1, 2, 3.

Special case $k = 0$. In this case, S defaults to CE model and $\tilde{\alpha}(k) = 0$. The last four cases in Table B2 do not exist. The rest of the above proof applies to CE by setting $k = 0$, and the results are captured in Proposition 1. □

Lemma B4. *Define function* $g(\alpha) = \frac{1}{16(1-\alpha)} - \frac{2\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2}$. *Then, on the interval* $(0, 13-4\sqrt{10})$, *equation* $g(\alpha) = 0$ *has a unique solution* $\underline{\alpha} \approx 0.065$. *Furthermore,* $g(\alpha) > 0$ *for* $\alpha \in (0, \underline{\alpha})$ *and* $g(\alpha) < 0$ *for* $\alpha \in (\underline{\alpha}, 13-4\sqrt{10})$.

Proof. We consider 3 cases:

(i) $\alpha \in \left(0, \frac{1}{32}\right)$. Then $g(\alpha) = \frac{\sqrt{1+\alpha}(1-32\alpha)+\sqrt{2\alpha}(1+32\alpha)}{16(1-\alpha)(\sqrt{1+\alpha}+\sqrt{2\alpha})} > 0$.

(ii) $\alpha \in \left[\frac{1}{32}, \frac{1}{4}\right]$. We can rewrite $g(\alpha)$ as

$$g(\alpha) = \frac{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2 - 32\alpha(1-\alpha)}{16(1-\alpha)\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2}. \tag{B.27}$$

We define $\psi(\alpha) \triangleq \left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2 - 32\alpha(1-\alpha)$. For $\alpha \in \left[\frac{1}{32}, \frac{1}{4}\right]$, we have $sign\{g(\alpha)\} = sign\{\psi(\alpha)\}$, and g and ψ have the same roots. We next prove that $\psi(\alpha)$ has a unique root $\underline{\alpha}$ in the interval $\left[\frac{1}{32}, \frac{1}{4}\right]$. We have:

$$\frac{\partial \psi}{\partial \alpha}(\alpha) = 2\left(\sqrt{\frac{1+\alpha}{2\alpha}} + \frac{1}{2}\sqrt{\frac{2\alpha}{1+\alpha}}\right) - 29 + 64\alpha.$$

Over interval $\left[\frac{1}{32}, \frac{1}{4}\right]$, it can be shown that the function $y(\alpha) \triangleq \sqrt{\frac{1+\alpha}{2\alpha}} + \frac{1}{2}\sqrt{\frac{2\alpha}{1+\alpha}}$ is maximized when $\alpha = \frac{1}{32}$, in which case $y\left(\frac{1}{32}\right) \approx 4.185$. From (B.28), we see that:

$$\frac{\partial \psi}{\partial \alpha}(\alpha) \leq 2 \times 4.185 - 29 + 64 \times \frac{1}{4} < 0, \qquad \forall\ \alpha \in \left[\frac{1}{32}, \frac{1}{4}\right]. \tag{B.28}$$

Moreover, $\psi\left(\frac{1}{32}\right) \approx 0.63$ and $\psi\left(\frac{1}{4}\right) \approx -2.67$. Therefore, it immediately follows that $\psi(\alpha) = 0$ has a unique solution $\underline{\alpha}$ in the interval $\left[\frac{1}{32}, \frac{1}{4}\right]$. Solving numerically the equation using Matlab we obtained $\underline{\alpha} \approx 0.065$. From (B.27) it immediately follows that $g(\underline{\alpha}) = 0$, $g(\alpha) > 0$ for $\alpha \in \left[\frac{1}{32}, \underline{\alpha}\right)$, and $g(\alpha) < 0$ when $\alpha \in \left(\underline{\alpha}, \frac{1}{4}\right]$.

(iii) $\alpha \in \left(\frac{1}{4}, 13-4\sqrt{10}\right)$. Here, $\frac{1}{16(1-\alpha)} < \frac{1}{8} < \frac{\alpha}{2} < \frac{2\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2}$. Thus, $g(\alpha) < 0$. □

Proof of Proposition 2. For any $k \in [0, 1)$, we have:

$$13 - 4\sqrt{10} \leq \frac{13 + 2k + k^2 - 4\sqrt{10 + 2k^2 + 4k}}{(1-k)^2} \leq \frac{1}{\sqrt{3+k^2} + 1 - k}. \tag{B.29}$$

We split the analysis into several cases:

Case 1. $13 - 4\sqrt{10} \leq \alpha$. Consider a fixed $k \in [0, 1)$. We have several subcases:

(i) $\frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2} < \alpha$. Then, $\pi_S^* = \frac{c\alpha(1-k)}{2} \leq \frac{c\alpha}{2} = \pi_{CE}^*$.

(ii) $13 - 4\sqrt{10} \leq \alpha \leq \frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2}$. It can be shown that:

$$\pi_S^* = \frac{2c\alpha(1-k)}{\left(\sqrt{2\alpha+(1-\alpha)(1-k)}+\sqrt{2\alpha}\right)^2} \leq \frac{2c\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2} \leq \frac{c\alpha}{2} = \pi_{CE}^*.$$

Reconciling cases (i) and (ii) we see that $\pi_S^* \leq \pi_{CE}^*$ for any $k \in [0, 1)$. Since the upper bound is attainable when $k = 0$, it follows that $k_S^* = 0, \ \forall \ \alpha \geq 13 - 4\sqrt{10}$.

Case 2. $\underline{\alpha} \leq \alpha < 13 - 4\sqrt{10}$. In this case, for any $k \in [0, 1)$, it can be shown that:

$$\frac{(1-k)c[\alpha+(1-\alpha)k]}{4} \leq \frac{c}{16(1-\alpha)} \leq \frac{2\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2} = \pi_{CE}^*.$$

Furthermore:

$$\frac{2c\alpha(1-k)}{\left(\sqrt{2\alpha+(1-\alpha)(1-k)}+\sqrt{2\alpha}\right)^2} \leq \frac{2c\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2} = \pi_{CE}^*.$$

From Lemma B3, regardless of where $\tilde{\alpha}(k)$ falls, we have $\pi_S^* \leq \pi_{CE}^*$. Since the upper bound is attainable when $k = 0$, we have $k_S^* = 0, \ \forall \ \alpha \in \left[\underline{\alpha}, 13 - 4\sqrt{10}\right)$.

Case 3. $0 < \alpha < \underline{\alpha}$. In this case, for any $k \in [0, 1)$, it can be shown that:

$$\frac{2c\alpha(1-k)}{\left(\sqrt{2\alpha+(1-\alpha)(1-k)}+\sqrt{2\alpha}\right)^2} \leq \frac{2c\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2} = \pi_{CE}^* < \frac{c}{16(1-\alpha)}. \tag{B.30}$$

We next show that the upper bound $\frac{c}{16(1-\alpha)}$ is attainable. Clearly, $\frac{(1-k)c[\alpha+(1-\alpha)k]}{4}$ is maximized at $k = \frac{1-2\alpha}{2-2\alpha}$ and $\left.\frac{(1-k)c[\alpha+(1-\alpha)k]}{4}\right|_{k=\frac{1-2\alpha}{2-2\alpha}} = \frac{c}{16(1-\alpha)}$. Thus, for any $k \in [0, 1)$:

$$\pi_S^* \leq \frac{c}{16(1-\alpha)}.$$

In order to verify that the upper bound is attained by setting $k_S^* = \frac{1-2\alpha}{2-2\alpha}$, all that is left to check is that $\tilde{\alpha}\left(\frac{1-2\alpha}{2-2\alpha}\right) > \underline{\alpha}$ due to Lemma B3. Plugging $k = \frac{1-2\alpha}{2-2\alpha}$ in $h_k(\alpha)$, we obtain:

$$h_k(\alpha) = \frac{1}{2}\left(\sqrt{2\alpha+\frac{1}{2}}+\sqrt{2\alpha}\right)^2 - 8\alpha. \tag{B.31}$$

The unique solution (implicitly equal to $\tilde{\alpha}\left(\frac{1-2\alpha}{2-2\alpha}\right)$) to equation $h_{\frac{1-2\alpha}{2-2\alpha}}(\alpha) = 0$ over the interval $(0, \infty)$ is $\tilde{\alpha}\left(\frac{1-2\alpha}{2-2\alpha}\right) = \frac{2+\sqrt{2}}{32} \approx 0.107 > \underline{\alpha} \approx 0.065$. Therefore, in this region, it immediately follows that:

$$k_S^* = \frac{1-2\alpha}{2-2\alpha}, \quad p_S^* = \frac{c}{4}, \quad \pi_S^* = \frac{c}{16(1-\alpha)},$$

and paid adoption occurs solely in period 2. $\square$

Proof of Proposition 3. Follows directly from the equilibria for each strategy and the properties of function f introduced at the beginning of §4.4. □

Corollary B1. *Under constant pricing, when* $\alpha > 13 - 4\sqrt{10}$*, then CE strategy is dominating iff* $\delta_b \le \frac{\alpha_a}{b}$.

Proof. When $\alpha > 13-4\sqrt{10}$, under CE adoption happens only in period 1 and $\pi_{CE} = \frac{(b+1)\alpha}{2}$. When $\delta_b > \frac{\alpha_a}{b}$, then $\alpha_b b + \delta_b b > \alpha_a + \alpha_b b$. Thus, $\frac{b}{b+1} \times (\alpha_b + \delta_b) > \alpha$. Thus $\alpha_b + \delta_b > 13 - 4\sqrt{10}$. Thus, under FLF, adoption also happens only in period 2 and $\pi_{FLF} = \frac{b(\alpha_b+\delta_b)}{2} > \frac{(b+1)\alpha}{2} = \pi_{CE}$. Thus, when $\alpha > 13 - 4\sqrt{10}$ and $\delta_b > \frac{\alpha_a}{b}$, then FLF dominates CE.

When $\delta_b \le \frac{\alpha_a}{b}$, it follows that $\frac{b}{b+1} \times (\alpha_b + \delta_b) \le \alpha$. If $\alpha_b + \delta_b \ge 13 - 4\sqrt{10}$, then the previous inequality directly translates into $\pi_{FLF} \le \pi_{CE}$. When $\alpha_b+\delta_b \ge 13-4\sqrt{10}$, then $\pi_{FLF} = bf(\alpha_b+\delta_b) < bf(13 - 4\sqrt{10}) = \frac{b(13-4\sqrt{10})}{2} < \frac{(b+1)\alpha}{2} = \pi_{CE}$. Thus, when $\alpha > 13 - 4\sqrt{10}$ and $\delta_b \le \frac{\alpha_a}{b}$, then CE dominates FLF.

The firm is indifferent between the two strategies when $\delta_b = \frac{\alpha_a}{b}$. □

Proposition B1. *Under CE and constant pricing, when* $w = \infty$*, firm's optimal pricing strategy is:*

	$0 < \alpha < \frac{1}{4}$	$\frac{1}{4} \le \alpha < \frac{1}{2}$	$\frac{1}{2} \le \alpha$
p^*_{CE}	$2c\alpha^-$	$\frac{c}{2}$	$c\alpha$
π^*_{CE}	$2c\alpha\,(1 - 2\alpha)$	$\frac{c}{4}$	$\frac{c\alpha}{2}$
Paid adoption	*in both periods (but in negligible volume in period 1)*	*in both periods*	*only in period 1*

Proof. For adoption to start (and WOM effects to disseminate), it is necessary to have $p \le 2c\alpha$ and the firm will only consider such strategies. Once adoption starts, we have $\theta_1 = \frac{p}{2c\alpha}$. However, unlike in the case of $w = 1$, when $w = \infty$ we have $c_2 = c$. Thus, $\theta_2 = min\left\{\theta_1, \frac{p}{c}\right\}$. It can be seen that adoption happens in period 2 if and only if $\theta_2 < \theta_1$, or $\alpha < \frac{1}{2}$. We distinguish three cases:

(i) $\frac{1}{2} \le \alpha$. Then adoption starts in period 1 but $\theta_2 = \theta_1$ and, thus, there is no additional adoption in period 2. In this case, $\pi_{CE} = p\left(1 - \frac{p}{2c\alpha}\right)$. Solving for optimal price yields $p^*_{CE} = c\alpha < 2c\alpha$. The constraint on price is not binding.

(ii) $\frac{1}{4} \le \alpha < \frac{1}{2}$. In this case, adoption happens in both periods. Thus, $\pi_{CE} = p\left(1 - \frac{p}{c}\right)$. Solving for optimal price yields $p^*_{CE} = \frac{c}{2} \le 2c\alpha$. The constraint on price is not binding.

(iii) $0 < \alpha < \frac{1}{4}$. Differently from case (ii), in this case, the constraint is binding because $\frac{c}{2} > 2c\alpha$. Thus $p^*_{CE} = 2c\alpha^-$. Adoption starts in period 1 in negligible volume. However, given that $w = \infty$, this is enough to inform all remaining potential customers about the true value of the product. □

Proposition B2. *Under S and constant pricing, when* $w = \infty$*, firm's optimal seeding and pricing strategies are:*

	$0 < \alpha < \frac{1}{4}$	$\frac{1}{4} \leq \alpha < \frac{1}{2}$	$\frac{1}{2} \leq \alpha$
k_S^*	0^+	0	0
p_S^*	$\frac{c}{2}$	p_{CE}^*	p_{CE}^*
π_S^*	$\frac{c}{4}$	π_{CE}^*	π_{CE}^*
Paid adoption	*only in period 2*	*in both periods*	*only in period 1*

where 0^+ *represents a negligible positive quantity.*

Proof. The firm will choose optimal strategy such that either we have paid adoption in period 1 or seeding (if neither occurs, there will be no paid adoption in period 2 either). This means that WOM effects kick in at the end of period 1 and $c_2 = c$. Moreover, the firm will price such that $p \leq \max\{2c\alpha, c\}$ because otherwise no paid adoption can occur. We have $\theta_1 = \min\left\{1, \frac{p}{2c\alpha}\right\}$ and $\theta_2 = \min\left\{\theta_1, \frac{p}{c}\right\}$.

Under optimal strategy, paid adoption occurs in period 2 if and only if $\alpha < \frac{1}{2}$. When $\alpha \geq \frac{1}{2}$, then $\frac{p}{2c\alpha} \leq \frac{p}{c}$ which means there cannot be paid adoption in period 2. When $\alpha < \frac{1}{2}$ we can have two cases: (a) $0 < p < 2c\alpha$, and (b) $2c\alpha \leq p < c$. In case (a) there is paid adoption in both periods. In case (b) there is paid adoption in period 2 but no paid adoption in period 1 (only seeding in period 1).

We break the analysis into two cases:

(i) $\frac{1}{2} \leq \alpha$. The price constraint is $p < 2c\alpha$ and paid adoption occurs only in period 1. Thus $\pi_S = p(1-k)\left(1 - \frac{p}{2c\alpha}\right)$. Solving for optimal price yields $p_S^* = c\alpha < 2c\alpha$. The constraint on price is not binding. There is no seeding in this case and the strategy defaults to the optimal strategy under CE.

(ii) $0 < \alpha < \frac{1}{2}$. The price constraint is $p < c$. In this case there is always paid adoption in period 2. Then $\pi_S = p(1-k)\left(1 - \frac{p}{c}\right)$. Solving for optimal price yields $p_S^* = \frac{c}{2} < c$. The constraint on price is not binding.

If $\alpha > \frac{1}{4}$ there is paid adoption in period 1 and thus there is no need for seeding since $w = \infty$ and thus more adopters in period 1 do not lead to any outcome difference in the valuation learning process.

If $\alpha \leq \frac{1}{4}$, then there is no paid adoption and the firm will seed a bare minimum of the market (just enough to spark WOM since then there is perfect learning in the market). Thus $k_S^* = 0^+$, a negligible positive quantity. □

Proof of Proposition 4. FLF is a particular case of CE substituting $c \to b$ and $\alpha \to \alpha_b + \delta_b$, as discussed in §4.2. The results follow immediately by direct profit comparison using the results in Propositions B1 and B2. □

C T Periods

As discussed in §4.7, the number of periods has a somewhat similar impact as w in essence. In other words, we see very similar forces at play when T increases (and w stays constant) compared to the case in §4.6 where w was increasing (and T was held constant at 2). For that reason, a brief discussion has been included in the main text in §4.7 and the corresponding analytical and numerical results supporting that discussion have been moved to Online Supplement C. In Figure C1, we see how an

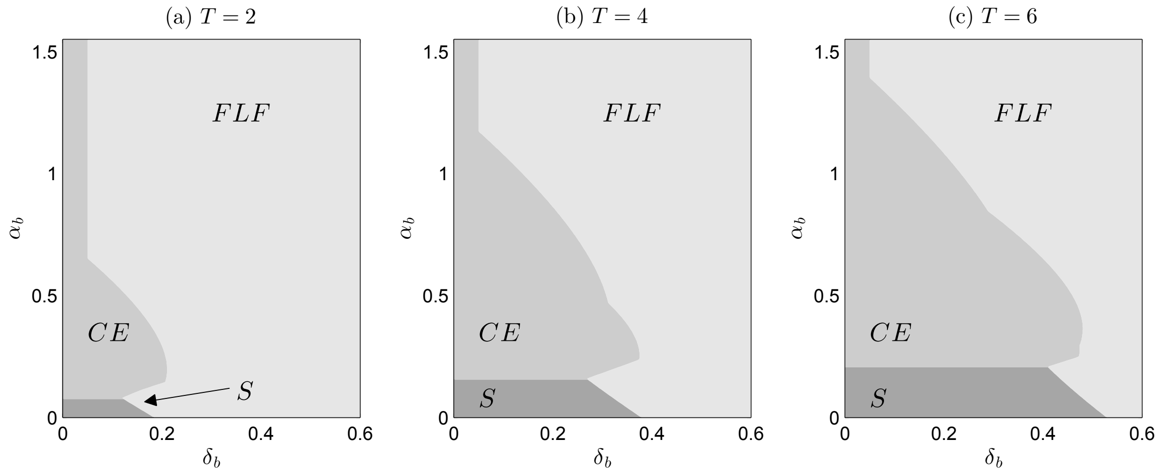

Figure C1: Firm's optimal strategies for a variable number of time periods ($w = 1$, $\alpha_a = 0.05$, $b = 1$).

increase in T generates similar effects as the ones observed in Figure 4 in association with an increase in w. Similar insights apply and we omit replicating the discussion.

We next explore the case of instantaneous dissemination of information ($T = \infty$). Such a case characterizes very efficient information transfer in the markets. Below we derive the equilibria under CE (and, implicitly, FLF) and S for $T = \infty$.

Proposition C1. *Consider CE with T periods, constant pricing, and lifetime value factor of the product fixed at 2c (i.e., $\frac{2c}{T}$ per period). When $T = \infty$ and w is finite, the firm's optimal pricing strategy is:*

	$0 < \alpha < \frac{1}{2}$	$\frac{1}{2} \leq \alpha < 1$	$1 \leq \alpha$
p^*_{CE}	$2c\alpha^-$	c	$c\alpha$
π^*_{CE}	$2c\alpha(1-\alpha)$	$\frac{c}{2}$	$\frac{c\alpha}{2}$

Proof. To get any adoption started, it must be the case that $p \leq 2c\alpha$. When $\alpha \geq 1$, customers either adopt at the very beginning or they never adopt (because under WOM they would always revise downwards their perceived valuation for the product). Then it can be show that $p^*_{CE} = c\alpha$.

When $0 < \alpha < 1$, once adoptions starts in period 1, customers learn very fast the value of the product and the type of the marginal adopter becomes $\theta_m = \frac{p}{2c}$ as relatively all value of the product is still accessible to the user at decision time. Then, the profit is approximately $p\left(1 - \frac{p}{2c}\right)$. In this region, optimal price is derived by solving the constrained profit maximization. □

Proposition C2. *Consider S with T periods, constant pricing, and lifetime value factor of the product fixed at 2c (i.e., $\frac{2c}{T}$ per period). When $T = \infty$ and w is finite, the firm's optimal seeding and pricing strategies are:*

	$0 < \alpha < \frac{1}{2}$	$\frac{1}{2} \leq \alpha$
k^*_S	0^+	0
p^*_S	c	p^*_{CE}
π^*_S	$\frac{c}{2}$	π^*_{CE}

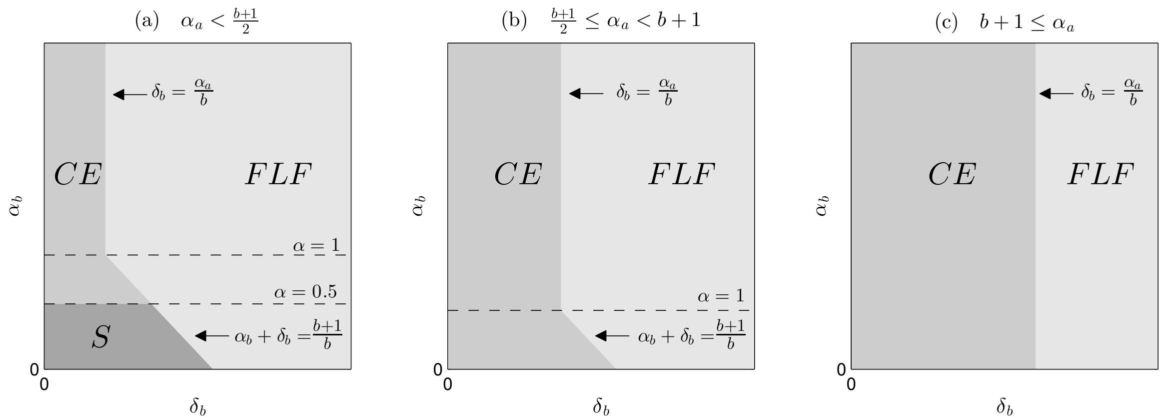

Figure C2: Firm's optimal strategies ($T = \infty$, w is finite).

Proof. When $\alpha \geq 1$, since WOM only induces users to downgrade their valuation, the firm has no incentives to seed the market. Thus $k_S^* = 0$ and the firm's strategy is basically CE. When $\alpha < 1$, then if there is seeding or paid adoption starts in period 1, customers quickly learn the value of product and the type of the marginal adopter becomes $\theta_m = \frac{p}{2c}$. Since $p \leq 2c$ in order to have any adoption, it follows that $\pi_S = p(1-k)\left(1 - \frac{p}{2c}\right)$. When $1 > \alpha \geq \frac{1}{2}$ then it is not necessary to seed the market since optimal price $p_S^* = c$ induces adoption in period 1 as well, which in turn induces everyone to learn very fast the true value of the product. When $\alpha < \frac{1}{2}$, the firm will seed a negligible portion of the market ($k_S^* = 0^+$) and $p_S^* = c$. □

In the next result and in Figure C2, we show how the parameter space is divided in optimality regions corresponding to the three strategies.

Proposition C3. *Consider a framework with T periods, constant pricing, and lifetime value factors of the two modules fixed at $2a$ and $2b$ (i.e., $\frac{2a}{T}$ and $\frac{2b}{T}$ per period) with a normalized to 1. When $T = \infty$ and w is finite, for any feasible set $\{\alpha_a, \alpha_b, \delta_b, b\}$, if the firm can choose among CE, FLF, and S business models, the following hold true:*

(a) if $\delta_b \geq \max\left\{\frac{b+1}{b} - \alpha_b, \frac{\alpha_a}{b}\right\}$ then FLF is the dominating strategy;

(b) if $0 < \alpha_a \leq \frac{b+1}{2}$, $\alpha_b \leq \frac{b+1}{2b} - \frac{\alpha_a}{b}$, and $\delta_b \leq \frac{b+1}{b} - \alpha_b$, then S is the dominating strategy and $k_S^* = 0^+$, where 0^+ denotes a negligible positive quantity;

(c) otherwise CE is the dominating strategy.

Proof. The results follow immediately by direct profit comparison using the results in Propositions C1 and C2 . □

D Dynamic Pricing with No Commitment and Myopic Customers

Suppose the firm has the ability to change the price in the second period. The firm charges p_1 for the product if it is bought in period 1 and p_2 if it is bought in period 2. Consider the case of moderate WOM effects ($w = 1$). We consider in this section the case where the firm does not commit to price (and does not announce p_2 in period 1) and customers act myopically by making a purchase decision in period 1 solely based on p_1 and without anticipating any change in price in period 2. In this case, the optimal pricing strategy can be solved by backwards induction as p_2 does not have an impact on the adoption outcome in period 1, and, thus, can be optimized by taking N_1 fixed at the end of period 1.

Proposition D1. *Under CE with dynamic pricing, no price commitment (and p_2 announced in period 2), myopic customers, when $w = 1$, the following equilibrium emerges:*

$$p^*_{CE,1} = \begin{cases} \frac{2c\alpha\left(8\alpha-1-\sqrt{40\alpha^2+8\alpha+1}\right)}{3(\alpha-1)}, & \alpha > 1 \\ \frac{8c}{7}, & \alpha = 1 \\ \frac{2c\alpha\left(\sqrt{40\alpha^2+8\alpha+1}-8\alpha+1\right)}{3(1-\alpha)}, & \alpha < 1 \end{cases}, \tag{D.1}$$

$$p^*_{CE,2} = \frac{p^*_{CE,1}\left(2c\alpha - p^*_{CE,1}(1-\alpha)\right)}{8c\alpha^2}, \tag{D.2}$$

$$\pi^*_{CE} = \frac{p^*_{CE,1}}{32c^2\alpha^3}\left(32c^2\alpha^3 + p^*_{CE,1}2c\alpha(1-8\alpha) - p^{*2}_{CE,1}(1-\alpha)\right). \tag{D.3}$$

Proof. The firm will always choose $p_1 \in (0, 2c_1)$. Moreover, given that the threshold adopter in period 1 is $\theta_1 = \frac{p}{2c_1}$ the firm will choose $p_2 \in \left(0, \frac{p_1 c_2}{2c_1}\right)$. Then, the number of adopters in periods 1 and 2 are $N_1 = 1 - \frac{p_1}{2c_1}$ and $N_2 - N_1 = \frac{p_1}{2c_1} - \frac{p_2}{c_2}$, and the profit is given by:

$$\pi_{CE}(p_1, p_2) = p_1\left(1 - \frac{p_1}{2c_1}\right) + p_2\left(\frac{p_1}{2c_1} - \frac{p_2}{c_2}\right). \tag{D.4}$$

Note that $c_2 = c_1 + N_1(c - c_1) = c_1 + \left(1 - \frac{p_1}{2c_1}\right)(c - c_1) = c - \frac{p_1(1-\alpha)}{2\alpha}$. It immediately follows that:

$$p^*_2(p_1) = \frac{p_1 c_2}{4c_1} = \frac{p_1\left(c - \frac{p_1(c-c_1)}{2c_1}\right)}{4c_1}. \tag{D.5}$$

Then the profit can be rewritten as $\pi_{CE}(p_1) = p_1 \times \ell(p_1)$, where $\ell(p_1) = 1 + p_1 \times \frac{(c-8c_1)}{16c_1^2} - p_1^2 \times \frac{(c-c_1)}{32c_1^3}$. We consider three cases:

Case 1: $c_1 \geq c$. In this case $\alpha > 1$. Then it can be shown that $\ell(p_1) = 0$ has two real positive roots $0 < p^\ell_{1,1} \leq p^\ell_{1,2}$. Then, $\pi_{CE}(p_1)$ is (i) increasing and then decreasing for $p_1 \in (0, p^\ell_{1,1}]$, (ii) decreasing and then increasing for $p_1 \in (p^\ell_{1,1}, p^\ell_{1,2}]$, and (iii) increasing for $p_1 \in (p^\ell_{1,2}, \infty)$. The FOC: $\pi'_{CE}(p_1) = 0$ has two roots:

$$p_{1,1} = \frac{2c_1\left(8c_1 - c - \sqrt{40c_1^2 + 8c_1c + c^2}\right)}{3(c_1 - c)} \quad \text{and} \quad p_{1,2} = \frac{2c_1\left(8c_1 - c + \sqrt{40c_1^2 + 8c_1c + c^2}\right)}{3(c_1 - c)}. \tag{D.6}$$

$p_{1,1}$ is the unique local maximum. Then it immediately follows that $p^*_1 = \min\{2c_1, p_{1,1}\}$. However, it can be easily shown that $p_{1,1} \leq 2c_1$ whenever $c_1 \geq c$. Thus, $p^*_{CE,1} = p_{1,1}$.

Case 2: $c = c_1$. In this case $\alpha = 1$ and the profit $\pi_{CE} = p_1 - \frac{7}{16c}p_1^2$ is quadratic and concave in p_1. Then $p^*_{CE,1} = \frac{8c}{7}$.

Case 3: $c > c_1$. In this case $\alpha < 1$. Then it can be shown that $\ell(p_1) = 0$ has two real roots $p^\ell_{1,1} < 0 < p^\ell_{1,2}$. Then, $\pi_{CE}(p_1)$ is (i) increasing and then decreasing for $p_1 \in (0, p^\ell_{1,2}]$ and (ii) decreasing for $p_1 \in (p^\ell_{1,2}, \infty]$. The FOC: $\pi'_{CE}(p_1) = 0$ has two roots $p_{1,2} < 0 < p_{1,1}$ with the solutions given in (D.6). Then, it immediately follows that $p_1^* = \min\{2c_1, p_{1,1}\}$. Moreover, it can be easily shown that $p_{1,1} \le 2c_1$ whenever $c > c_1$. Consequently $p_1^* = p_{1,1}$.

This completes the proof. □

Lemma D1. *Under S and similar assumptions as in Proposition D1, for a given seeding ratio* $k \in [0, 1)$, *the firm's optimal strategy is:*

$$p^*_{S,1}(k) = \begin{cases} \frac{2c\alpha\left(8\alpha-1-\sqrt{(40+24k)\alpha^2+(8-24k)\alpha+1}\right)}{3(\alpha-1)(1-k)}, & \alpha > 1, \\ \frac{8c}{7}, & \alpha = 1, \\ \frac{2c\alpha\left(\sqrt{(40+24k)\alpha^2+(8-24k)\alpha+1}-8\alpha+1\right)}{3(1-\alpha)(1-k)}, & \max\{0, \frac{3k-1}{5+3k}\} \le \alpha < 1, \\ 2c\alpha & 0 < \alpha < \frac{3k-1}{5+3k} \ \ \textit{and} \ \ \frac{1}{3} \le k < 1, \end{cases}$$

$$p^*_{S,2}(k) = \frac{p^*_{S,1}\left(2c\alpha - p^*_{S,1}(1-\alpha)(1-k)\right)}{8c\alpha^2},$$

$$\pi^*_S(k) = \frac{(1-k)p^*_{S,1}}{32c^2\alpha^3}\left(32c^2\alpha^3 + 2c\alpha(1-8\alpha)p^*_{S,1} - (1-k)(1-\alpha)p^{*2}_{S,1}\right).$$

Proof. The firm will always choose $p_1 \in (0, 2c_1]$. Unlike in the CE case, the firm can actually choose $p_1 = 2c_1$ such that there is no paid adoption in period 1 because it can still induce WOM effects via seeding (outcome is the same when $p_1 > 2c_1$ and, for simplicity, we assume the firm is not considering those). Then, $c_2 = c_1 + (N_1 + k)(c - c_1)$. Similar to CE case, we can immediately derive $\theta_1 = \frac{p}{2c_1}$, $\theta_2 = \frac{p_2 c1}{c_2}$, and $p_2^* = \frac{p_1 c_2}{4c_1} = \frac{p_1[2c\alpha - p_1(1-k)(1-\alpha)]}{8c\alpha^2}$. Then profit is given by:

$$\begin{aligned} \pi_S(p_1, p_2) &= p_1(1-\theta_1) + p_2(\theta_1 - \theta_2) \\ &= \frac{(1-k)p_1}{32c^2\alpha^3}\left(32c^2\alpha^3 + p_1 2c\alpha(1-8\alpha) - p_1^2(1-k)(1-\alpha)\right). \end{aligned}$$

Following a similar approach as in the case of CE, we obtain the desired results by choosing p_1^* that maximizes π_S subject to constraint $p_1 \le 2c\alpha$, where equality is possible this time. □

Proposition D2. *Under S and similar assumptions as in Proposition D1, the firm's optimal pricing and seeding strategy is given by:*

	$0 < \alpha < \underline{\hat{\alpha}}$	$\underline{\hat{\alpha}} \le \alpha$
k^*_S	$\frac{1-2\alpha}{2-2\alpha}$	0
$p^*_{S,1}$	$2c\alpha$	$p^*_{CE,1}$
$p^*_{S,2}$	$\frac{c}{4}$	$p^*_{CE,2}$
π^*_S	$\frac{c}{16(1-\alpha)}$	π^*_{CE}
Paid adoption	*only in period 2*	*in both periods*

where $\underline{\hat{\alpha}} \approx 0.0557$ *is the unique solution to equation* $\hat{g}(\alpha) = 0$ *over the interval* $\left[0, \frac{1}{16}\right)$*, with* $\hat{g}(\alpha) = \frac{1}{16(1-\alpha)} - \frac{1+12\alpha-132\alpha^2-224\alpha^3+(1+8\alpha+40\alpha^2)^{1.5}}{54(1-\alpha)^2}$.

Proof. Using Lemma D1, we explore when it is feasible under optimum to have $k > 0$.

(i) $\frac{1}{16} \leq \alpha$. In this case, it can be shown that $\pi_S^*(k, \alpha)$ is decreasing in k for $k \in [0, 1)$. Thus, $k_S^* = 0$ and S simplifies to CE.

(ii) $0 < \alpha < \frac{1}{16}$. Note that $\frac{3k-1}{5+3k} = \frac{1}{16}$ when $k = \frac{7}{15}$. We have several cases:

  (a) $0 \leq k < \frac{7}{15}$.

    (1) $\frac{3k-1}{5+3k} < \alpha < \frac{1}{16}$. In this case it can be shown that π_S^* is convex in k. Thus the firm will choose k at the boundary. So either $k_S^* = 0$ or we move to case (ii.a.2) because increasing k we first encounter boundary $\frac{3k-1}{5+3k} = \alpha$ before encountering boundary $\frac{3k-1}{5+3k} = \frac{1}{16}$ at $k = \frac{7}{15}$.

    (2) $0 < \alpha \leq \frac{3k-1}{5+3k}$. This case occurs only when $\frac{1}{3} < k < \frac{7}{15}$. In this case, it can be shown that $\pi_S^*(k, \alpha)$ is increasing in k. This case cannot be optimal. By increasing k, we move to case (ii.b).

  (b) $\frac{7}{15} \leq k < \frac{1}{2}$. Then, in this case, $\alpha < \frac{3k-1}{5+3k}$. Then, it can be shown that, under S, the firm will choose optimal $k_S^* = \frac{1-2\alpha}{2-2\alpha}$, which yields $\pi_S^*(\alpha) = \frac{c}{16(1-\alpha)}$. In this case, $\pi_S^*(\alpha) - \pi_{CE}^*(\alpha) = c\left[\frac{1}{16(1-\alpha)} - \frac{1+12\alpha-132\alpha^2-224\alpha^3+(1+8\alpha+40\alpha^2)^{1.5}}{54(1-\alpha)^2}\right] = c \times \hat{g}(\alpha)$. Then, it can be easily shown that $\hat{g}(\alpha)$ is decreasing for $\alpha \in [0, \frac{1}{16}]$ with $\hat{g}(0) > 0 > \hat{g}(\frac{1}{16})$. We denote by $\underline{\hat{\alpha}}$ the unique solution to the equation $\hat{g}(\alpha) = 0$ over the interval $[0, \frac{1}{16}]$. We have $\underline{\hat{\alpha}} \approx 0.0557$. Then S dominates CE for $\alpha \in (0, \underline{\hat{\alpha}})$ and reduces to CE (i.e., $k_S^* = 0$) when $\alpha \geq \underline{\hat{\alpha}}$.

  (c) $\frac{1}{2} < k < 1$. Then, in this case, $\alpha < \frac{3k-1}{5+3k}$. In this case, it can be shown that $\pi_S^*(k, \alpha)$ is decreasing in k. This cannot be optimal. By decreasing k, we move to case (ii.b). □

E Dynamic Pricing with Commitment and Consumer Bounded Rationality

Proof of Proposition 5. (a) When $\alpha > 1$, regardless of w, it is always the case that $c_2 \leq c_1$. Under $p_1(\alpha) = c_1$ and $p_2(\alpha) \geq \frac{c_1}{2}$, IC constraint is not binding because IR is satisfied for a customer of type $\theta = \frac{1}{2}$. IC is evaluated in period 1, when customers do not anticipate any change in their perceived valuation of the product. Thus, under our bounded rationality assumptions, a customer of type θ compares $2\theta c\alpha - p_1(\alpha)$ versus $\theta c\alpha - p_2(\alpha)$. Thus, when $\theta = \frac{1}{2}$, IC can only be marginally binding $\frac{1}{2} \times 2c\alpha - p_1(\alpha) = 0 \geq \frac{1}{2} \times \alpha - p_2(\alpha)$. Moreover, for any $\theta < \frac{1}{2}$, $c_2\theta - p_2(\alpha) < c_1\theta - \frac{c\alpha}{2} < 0$. Thus, no additional customers adopt in period 2. Under this strategy, $\pi_{CE}^*(\alpha) = \frac{c\alpha}{2}$.

Given that the above set of strategies yield the highest profit under no adoption in period 2, the only way in which the firm could potentially get a higher profit would be to induce period 2 adoption. We will prove by contradiction that this is impossible. Suppose we have a strategy (p_1, p_2) that yields period 2 adoption. Then IR is binding for period 2. Moreover, note that if there is no adoption in period 1 due to IC constraints, then $c_1 = c_2$ and the maximum the firm can make in period 2 is $\frac{c_1}{4}$ because the product can only be used for one period, which is again a dominated outcome. Therefore, we only need to focus on strategies that induce *adoption in both periods*. We have two cases: (i) IC is binding in period 1 and IR is binding in period 2, or (ii) IC is not binding in period 1 and IR is binding in both periods. We only need to consider $p_1 \leq 2c\alpha$ since any price level above this will not yield any adoption in period 1. We discuss the two cases separately:

(i) IC is binding in period 1, IR is binding in period 2, and adoption occurs in both periods. Then $\theta_1 = \frac{p_1 - p_2}{c\alpha}$ and $\theta_2 = \frac{p_2}{c_2}$. Moreover, IR needs to hold for θ_1, i.e., $\frac{p_1-p_2}{c\alpha} > \frac{p_1}{2c\alpha}$ or $p_1 \geq 2p_2$. We also need $\theta_1 \geq \theta_2$ or $p_1 \geq p_2\left(1 + \frac{c_1}{c_2}\right)$. Since $c_1 \geq c_2$, this implicitly leads to IR being satisfied for θ_1 in period 1. Moreover, since we need $\theta_1 \leq 1$, it is necessary that $p_1 \leq p_2 + c_1$. The profit maximization becomes:

$$\begin{aligned}
&\max_{p_1, p_2} \pi_{CE}(p_1, p_2) = p_1\left(1 - \frac{p_1 - p_2}{c_1}\right) + p_2\left(\frac{p_1 - p_2}{c_1} - \frac{p_2}{c_2}\right), \\
&s.t. \\
&p_1 \in [0, 2c_1),\ \ p_2 \geq 0, \\
&p_2 + c_1 \geq p_1 \geq p_2\left(1 + \frac{c_1}{c_2}\right).
\end{aligned}$$

In the above optimization, c_2 is a function of p_1 that always satisfies $c_2 \leq c_1$. Simplifying the problem, for any set of *constants* $c_2 < c_1$, it can be easily shown that the above maximization problem can never yield a profit above $\frac{c_1}{2}$. As such, setting c_2 at $c_2(p_1^*)$, the above maximization problem will still yield a dominated outcome in the best scenario.

(ii) IC is not binding in period 1, IR is binding in both periods, and adoption occurs in both periods. Then $\theta_1 = \frac{p_1}{2c_1}$ and $\theta_2 = \frac{p_2}{c_2}$. IC not binding in period 1 implies $0 = 2\theta_1 c_1 - p_1 > \theta_1 c_1 - p_2$, or $p_1 < 2p_2$. At the same time, adoption in period 2 implies $\frac{p_1}{2c_1} > \frac{p_2}{c_2}$ or $p_1 > \frac{2c_1}{c_2} \times p_2 \geq 2p_2$ since $c_1 \geq c_2$. Thus, we reach a contradiction. Such a scenario can only happen when $\alpha < 1$.

In summary, it is optimal to price $p_1^*(\alpha) = c\alpha$ and $p_2^*(\alpha) \geq \frac{c\alpha}{2}$.

(b) Consider $\alpha \in \left[\frac{1}{2}, 1\right]$. If there is no adoption in period 1 (due to high p_1), then $c_1 = c_2$ and, at best, the firm can cover half of the market in period 2 with price $p_2 = \frac{c_1}{2}$ and profit $\frac{c_1}{4}$. On the other hand, if the firm prices at $p_1 = c_1$ and $p_2 = c > c_2 \geq c_1$, then there is no adoption in period 2, the firm covers half of the market in period 1 and makes profit $\frac{c_1}{2}$. Thus, under optimal pricing, there are sales in period 1.

Suppose that it is not optimal for IC to be binding in period 1. Then IR is binding in period 1. When $\alpha \in \left[\frac{1}{2}, 1\right]$, then $\frac{c_2}{c_1} = 1 + N_1^{1/w}\left(\frac{1}{\alpha} - 1\right) > 1$. Then $\theta_1 = \frac{p_1}{2c_1}$ and $\theta_2 = \min(\theta_1, \frac{p_2}{c_2})$. If IC is not binding under optimality, then this means the firm in period 2 prices such that exactly half of the remaining market $[0, \theta_1]$ adopts, i.e., $p_2 = \frac{p_1 c_2}{4c_1}$. IC not binding in period 1 implies $0 = 2\theta_1 c_1 - p_1 > \theta_1 c_1 - p_2$, or $p_1 < 2p_2$. Thus, $p_1 < \frac{p_1 c_2}{2c_1}$, or $2c_1 < c_2$. Thus, we need $2 < \frac{c_2}{c_1} = 1 + N_1^{1/w}\left(\frac{1}{\alpha} - 1\right)$, or $1 < N_1^{1/w}\left(\frac{1}{\alpha} - 1\right)$. Since $N_1^{1/w} \leq 1$, we need $\frac{1}{\alpha} \geq 2$ or $\alpha < \frac{1}{2}$. Thus, we reach contradiction. Therefore, when $\alpha \in \left[\frac{1}{2}, 1\right]$, then *IC is always binding.*

Since IC is binding in period 1, given that customers update valuation upwards at the end of period 1 when $\alpha \leq 1$, then IR must be binding in period 2 (types immediately below the marginal type in period 1 would have positive utility in period 2). We have $\theta_1 = \frac{p_1 - p_2}{c_1}$ and $\theta_2 = \frac{p_2}{c_2}$. Since we want $1 \geq \theta_1 \geq \theta_2$, we need to have $p_2 + c_1 \geq p_1 \geq p_2\left(1 + \frac{c_1}{c_2}\right)$. Moreover, $2c_1\theta_1 \geq p_1$ or $p_1 \geq 2p_2$. Since $c_2 \geq c_1$ we have $2 \geq 1 + \frac{c_1}{c_2}$, and, thus, $p_2 + c_1 \geq p_1 \geq 2p_2$, $p_1 \in (0, 2c_1)$, and $p_2 \geq 0$ are the only constraints required. The optimization becomes:

$$\max_{p_1,p_2} \pi_{CE}(p_1,p_2) = p_1\left(1-\frac{p_1-p_2}{c_1}\right)+p_2\left(\frac{p_1-p_2}{c_1}-\frac{p_2}{c_2(p_1,p_2)}\right), \tag{E.1}$$
$$s.t.$$
$$p_1 \in [0,2c_1), p_2 \geq 0,$$
$$p_2 + c_1 \geq p_1 \geq 2p_2.$$

Note that c_1 is independent of p_1 and p_2. On the other hand c_2 depends on p_2. Note that, since $\alpha \in \left[\frac{1}{2},1\right]$, $c_1 \leq c_2 \leq c \leq 2c_1$. If c_2 were independent of p_2, then, for any constant $\tilde{c}_2 \in (c_1,c)$, after solving for p_2 in the constrained profit maximization with $\tilde{c}_2$ instead of $c_2(p_1,p_2)$, we would always obtain $p_2^*(p_1,c_1,\tilde{c}_2) = \frac{p_1}{2}$. Moreover, note that $\pi_{CE}(p_1,p_2,c_1,\tilde{c}_2)$ is increasing in $\tilde{c}_2$ when the other parameters are held constant. Now, for any $p_2 \leq \frac{p_1}{2}$ satisfying the constraints, it follows that $\pi_{CE}\left(p_1,p_2,c_1,c_2(p_1,p_2)\right) \leq \pi_{CE}\left(p_1,\frac{p_1}{2},c_1,c_2(p_1,p_2)\right)$. Moreover, note that for fixed p_1 and p_2, since $p_2 \leq \frac{p_1}{2}$, it can be immediately seen that $c_2(p_1,p_2) \leq c_2(p_1,\frac{p_1}{2})$. A higher price in period 2 induces more customers to adopt in period 1, hence also strengthening the WOM effects and bringing c_2 closer to c. Thus, $\pi_{CE}\left(p_1,\frac{p_1}{2},c_1,c_2(p_1,p_2)\right) \leq \pi_{CE}\left(p_1,\frac{p_1}{2},c_1,c_2(p_1,\frac{p_1}{2})\right)$. Consequently, at optimum, it must be the case that $p_2^*(p_1,c_1) = \frac{p_1}{2}$. Note that, in this case, IR and IC constraints are *simultaneously* binding in period 1.

(c-d) We prove (c) and (d) together by focussing on proving (d) - if $\alpha^{\ddagger} < \frac{1}{2}$ exists, we can always take $\alpha^{\dagger} = \alpha^{\ddagger}$ since $p_2 > p_1$ always implies that IC is not binding. Consider α very small. Given that $p_1 \leq 2c\alpha$, if $p_2 \leq p_1$, then, even if all consumers eventually purchase, the profit is upper bounded by $2c\alpha$. Thus, such a strategy with $p_1 \geq p_2$ would always yield negligible profits for small values of α.

Consider, a pricing strategy where $\tilde{p}_1 = 2(1-\tau)\alpha c$ with $\tau \in (0,1)$ and $\tilde{p}_2 = \frac{c(1-\tau)\tau^{1/w}}{2}$. When α is very low, clearly $\tilde{p}_2 > \tilde{p}_1$ and IC is not binding since even the highest type customer would not consider delaying under our bounded rationality assumptions. Thus, $\theta_1 = (1-\tau)$ and $N_1 = \tau$. Consequently $c_2 = c\alpha + \tau^{1/w}c(1-\alpha) = c\alpha(1-\tau^{1/w}) + c\tau^{1/w} \geq c\tau^{1/w}$. Then, $\theta_2 = \frac{\tilde{p}_2}{c_2} < \frac{\tilde{p}_2}{c\tau^{1/w}} = \frac{1-\tau}{2}$. Thus, for low enough α, under this strategy, $\theta_1 - \theta_2 > \frac{1-\tau}{2}$ customers adopt at price $\tilde{p}_2 = \frac{(1-\tau)\tau^{1/w}c}{2}$, which yields a lower bound $\frac{c(1-\tau)^2\tau^{1/w}}{4}$ for the profit. Since, as discussed above, best attainable profit under price skimming strategies $p_1 \geq p_2$ is negligible when α is low, clearly, in such ranges where customers severely underestimate the value of the product it is optimal to employ penetration pricing strategies ($p_1 < p_2$). □

Proof of Proposition 6. The solution for $R3$ region ($\alpha > 1$) is given by part (a) of Proposition 5.

For the remaining part of the proof we focus on $\alpha \in (0,1]$. Note that the solution derived in Proposition D1 under no commitment and myopic customers is also optimal under commitment and customers with bounded rationality as long as IC constraint is not binding in period 1. This happens when the marginal adopter in period 1 would not have an incentive to delay for period 2 (again under the assumption that customers do not anticipate a change in their beliefs in the value of the product), i.e., $2c_1\theta_1 - p_1 = 0 > c_1\theta_1 - p_2$. This translates into $0 > \frac{p_1}{2} - p_2$. Let $\tilde{p}_1$ and $\tilde{p}_2$ be the optimal solutions derived in Proposition D1. Then, $\tilde{p}_1$ and $\tilde{p}_2$ are optimal under bounded rationality and price commitment if and only if $\frac{1}{2} < \frac{\tilde{p}_2}{\tilde{p}_1} = \frac{2+8\alpha-\sqrt{40\alpha^2+8\alpha+1}}{12\alpha} \iff \alpha < \frac{1}{2\sqrt{3}}$. Thus, in $R1$ Region, when $\alpha \in \left(0,\frac{1}{2\sqrt{3}}\right)$ we recover the same optimal solution as in Proposition D1.

For $R2$ region, it must be the case that IC is binding in period 1 and IR is binding in period 2. Following an argument similar to the one in the proof of part (b) of Proposition 5, in this scenario we have $p_1 = 2p_2$. Then, IR and IC are simultaneously binding in period 1 because $2c_1\theta_1 - 2p_2 = c_1\theta_1 - p_2$, and this can only happen when $c_1\theta_1 = p_2$. Moreover, $c_2 = c_1 + \left(1 - \frac{p_1}{2c_1}\right)(c - c_1)$. Then, following the same derivation as in part (b) of Proposition 5, profit is given by (E.1) and can be rewritten in terms of p_1 as:

$$\pi_{CE}(p_1) = \frac{p_1\left(2c\alpha - p_1\right)\left(4c\alpha - p_1(1-\alpha)\right)}{4c\alpha\left(2c\alpha - p_1(1-\alpha)\right)}. \tag{E.2}$$

It can be shown that $\frac{\partial^2 \pi_{CE}(p_1)}{\partial p_1^2} < 0$ when $c > 0$, $\alpha \in \left[\frac{1}{2\sqrt{3}}, 1\right]$, and $p_1 \in [0, 2c\alpha]$. Thus, over region $R2$, π_{CE} is concave in p_1. Moreover, $\left.\frac{\partial \pi_{CE}(p_1)}{\partial p_1}\right|_{p_1=0} = 1 > 0$ and $\left.\frac{\partial \pi_{CE}(p_1)}{\partial p_1}\right|_{p_1=2c\alpha} = -\frac{1+\alpha}{2\alpha} < 0$. Thus, there exists a unique maximizing value $p_1 \in (0, 2c\alpha)$ which is interior and must satisfy FOC. Given that $\frac{\partial \pi_{CE}(p_1)}{\partial p_1} = \frac{\phi_{c,\alpha}(p_1)}{2c\alpha(2c\alpha - p_1(1-\alpha))^2}$, the result follows immediately. □

Lemma E1. *Under S, price commitment, consumer bounded rationality, when consumers initially undervalue the product ($\alpha \in (0,1)$), for a given seeding ratio $k \in [0,1)$ the firm's optimal strategy is as follows:*

(a) When $0 < \alpha < \frac{2k+\sqrt{3+2k+4k^2}}{2(3+2k)}$, then:

$$p^*_{S,1}(k) = \begin{cases} \frac{2c\alpha\left(\sqrt{(40+24k)\alpha^2+(8-24k)\alpha+1}-8\alpha+1\right)}{3(1-\alpha)(1-k)}, & \max\{0, \frac{3k-1}{5+3k}\} \le \alpha < \frac{2k+\sqrt{3+2k+4k^2}}{2(3+2k)}, \\ 2c\alpha & 0 < \alpha < \frac{3k-1}{5+3k} \text{ and } \frac{1}{3} \le k < 1, \end{cases}$$

$$p^*_{S,2}(k) = \frac{p^*_{S,1}\left(2c\alpha - p^*_{S,1}(1-\alpha)(1-k)\right)}{8c\alpha^2},$$

$$\pi^*_S(k) = \frac{(1-k)p^*_{S,1}}{32c^2\alpha^3}\left(32c^2\alpha^3 + 2c\alpha(1-8\alpha)p^*_{S,1} - (1-k)(1-\alpha)p^{*2}_{S,1}\right).$$

(b) When $\frac{2k+\sqrt{3+2k+4k^2}}{2(3+2k)} \le \alpha < 1$, then:

$$p^*_{S,1}(k) = \tilde{\mu}_{c,\alpha}(k), \quad p^*_{S,2}(k) = \frac{p^*_{S,1}(k)}{2},$$

$$\pi^*_S(k) = \frac{(1-k)p^*_{S,1}\left((1-k)(1-\alpha){p^*_{S,1}}^2 - 2c\alpha[3-2k(1-\alpha)-\alpha]p^*_{S,1} + 8c^2\alpha^2\right)}{4c\alpha[2c\alpha - p^*_{S,1}(1-k)(1-\alpha)]},$$

where $\tilde{\mu}_{c,\alpha}(k)$ is the unique solution over the interval $\left(c\alpha, \frac{4c\alpha}{3}\right)$ to the equation $\lambda_{c,\alpha}(\mu, k) = 0$, with

$$\lambda_{c,\alpha}(\mu,k) \triangleq -(1-k)^2(1-\alpha)^2\mu^3 + c\alpha(1-k)(1-\alpha)[6-2k(1-\alpha)-\alpha]\mu^2 - 4c^2\alpha^2[3-2k(1-\alpha)-\alpha]\mu + 8c^3\alpha^3.$$

Proof. The solution derived in Proposition D1 in a scenario of myopic customers and no price commitment is optimal under bounded rationality and price commitment if IC constraint is not binding in period 1, which can be shown to be equivalent to $p_1 < 2p_2$ (via an argument similar to the one in the proof of Proposition 6). Let $\hat{p}_1$ and $\hat{p}_2$ be the optimal solutions derived in Proposition D1. Then, $\hat{p}_1$ and $\hat{p}_2$ are optimal under bounded rationality and price commitment if and only if $\frac{1}{2} < \frac{\hat{p}_2}{\hat{p}_1} = \frac{2c\alpha - \hat{p}_1(1-\alpha)(1-k)}{8c\alpha^2} \iff 0 < \alpha < \frac{2k+\sqrt{3+2k+4k^2}}{2(3+2k)}$. Note that when $k < 1$ then $\max\left\{0, \frac{3k-1}{5+3k}\right\} < \frac{2k+\sqrt{3+2k+4k^2}}{2(3+2k)} < 1$. Hence, we have the optimal solution for case (a).

When $\frac{2k+\sqrt{3+2k+4k^2}}{2(3+2k)} \leq \alpha < 1$, then IC is binding in period 1 and IR is binding in period 2. Then, following a similar argument as in the proof of part (b) of Proposition 5, the profit maximization becomes:

$$\max_{p_1,p_2} \pi_S(p_1,p_2,k) = p_1(1-k)\left(1-\frac{p_1-p_2}{c_1}\right)+p_2(1-k)\left(\frac{p_1-p_2}{c_1}-\frac{p_2}{c_2(p_1,p_2)}\right), \tag{E.3}$$
$$s.t.$$
$$p_1 \in [0,2c_1), p_2 \geq 0,$$
$$p_2 + c_1 \geq p_1 \geq 2p_2,$$

where $c_2(p_1,p_2) = c_1 + \left[k+(1-k)\left(1-\frac{p_1-p_2}{c_1}\right)\right](c-c_1) > c_1$. Following a similar argument as in the proof of part (b) of Proposition 5, we obtain $p_2^* = \frac{p_1^*}{2}$. Replacing in the profit function, we obtain:

$$\pi_S^*(p_1,k) = \frac{(1-k)p_1\left((1-k)(1-\alpha)p_1^2 - 2c\alpha[3-2k(1-\alpha)-\alpha]p_1 + 8c^2\alpha^2\right)}{4c\alpha[2c\alpha - p_1(1-k)(1-\alpha)]}. \tag{E.4}$$

It can be shown that $\frac{\partial^2 \pi_S(p_1,k)}{\partial p_1^2} < 0$ when $c > 0$, $\alpha \in \left[\frac{2k+\sqrt{3+2k+4k^2}}{2(3+2k)}, 1\right)$, and $p_1 \in [0, 2c\alpha]$. Thus, over the region of interest, π_S is concave in p_1. Moreover, $\left.\frac{\partial \pi_S(p_1,k)}{\partial p_1}\right|_{p_1=0} = 1-k > 0$ and $\left.\frac{\partial \pi_S(p_1)}{\partial p_1}\right|_{p_1=2c\alpha} = -\frac{\alpha(1-k^2+(1-k)^2\alpha)}{2(k+\alpha-k\alpha)^2} < 0$. Thus, there exists a unique maximizing value $p_1 \in (0, 2c\alpha)$ which is interior and must satisfy FOC. Given that $\frac{\partial \pi_S(p_1)}{\partial p_1} = \frac{(1-k)\lambda_{c,\alpha}(p_1,k)}{2c\alpha(2c\alpha - p_1(1-\alpha)(1-k))^2}$, it immediately follows that $p_1^*(k)$ exists, is interior, and is uniquely defined by $\lambda_{c,\alpha}(p_1^*(k),k) = 0$. Moreover, it can be shown that $\left.\frac{\partial \pi_S(p_1,k)}{\partial p_1}\right|_{p_1=c\alpha} > 0 > \left.\frac{\partial \pi_S(p_1,k)}{\partial p_1}\right|_{p_1=\frac{4c\alpha}{3}}$. Thus, $p_1^* \in \left(c\alpha, \frac{4c\alpha}{3}\right)$. $\square$

Proof of Proposition 7. We break down the analysis in several cases based on the value of α.

Case 1. $\alpha \geq 1$. In this case, WOM effects can only lower consumer per-period valuation of the product. As such, the firm does not have any incentive to seed ($k^* = 0$) and its optimal strategy is the same as the one under CE.

Case 2. $\frac{1}{2\sqrt{3}} < \alpha < 1$. We break this case into two subcases:

(i) (a) $\frac{1}{2} \leq \alpha < 1$ or (b) $\frac{1}{2\sqrt{3}} < \alpha < \frac{1}{2}$ and $\frac{12\alpha^2-1}{8\alpha(1-\alpha)} < k < 1$. In both of these subcases we have $\frac{2k+\sqrt{3+2k+4k^2}}{2(3+2k)} < \alpha < 1$. Then, from Proposition E1, we see that $p_1^*(k) \in \left(c\alpha, \frac{4c\alpha}{3}\right)$ is given in implicit form as $\lambda_{c,\alpha}(p_1^*(k),k) = 0$, which can also be rewritten as:

$$p_1^{*3} = \frac{c\alpha(1-k)(1-\alpha)(6-2k(1-\alpha)-\alpha)p_1^{*2} - 4c^2\alpha^2(3-2k(1-\alpha)-\alpha)p_1^* + 8c^3\alpha^3}{(1-k)^2(1-\alpha)^2}. \tag{E.5}$$

Moreover, using implicit function theorem, it can be shown that:

$$\begin{aligned}\frac{dp_1^*}{dk} &= -\frac{\frac{\partial \lambda_{c,\alpha}(p_1^*(k),k)}{\partial k}}{\frac{\partial \lambda_{c,\alpha}(p_1^*(k),k)}{\partial p_1}} \\ &= \frac{(1-\alpha)p_1^*\left[8c^2\alpha^2 - c\alpha(8-4k(1-\alpha)-3\alpha)p_1^* + 2(1-k)(1-\alpha)p_1^{*2}\right]}{[2c\alpha(3-2k(1-\alpha)-\alpha) - 3(1-k)(1-\alpha)p_1^*][2c\alpha-(1-k)(1-\alpha)p_1^*]}.\end{aligned} \tag{E.6}$$

We are interested in the monotonicity of π_S with respect to k. We have $\frac{d\pi_S}{dk} = \frac{\partial \pi_S}{\partial p_1} \times \frac{dp_1^*}{dk} + \frac{\partial \pi_S}{\partial k}$.

Using the formula for π_S derived in part (b) of Lemma E1, replacing $\frac{dp_1^*}{dk}$ with (E.6), and using (E.5) to reduce the degree of the expressions in p_1, we obtain:

$$\frac{d\pi_S}{dk} = \frac{X(p_1^*(k), k, \alpha, c)}{Y(p_1^*(k), k, \alpha, c)}, \tag{E.7}$$

with

$$\begin{aligned}
X(p_1, k, \alpha, c) &= A(k, \alpha, c) \times B(p_1, k, \alpha, c),\\
A(\cdot) &= -\frac{c^4\alpha^4}{(1-k)(1-\alpha)},\\
B(\cdot) &= C_0(k, \alpha, c) + C_1(k, \alpha, c)p_1 + C_2(k, \alpha)p_1^2,\\
C_0(\cdot) &= 8c^2\alpha^2\left[(2k(1-\alpha) + 3\alpha)(2k(1-\alpha) + \alpha)(2 - 2k(1-\alpha) - \alpha) - 12\alpha\right],\\
C_1(\cdot) &= 4c\alpha(36\alpha + (2k(1-\alpha) + \alpha)[-8k^3(1-\alpha)^3 + 4k^2(1-\alpha)^2(4 - 5\alpha)\\
&\qquad -2k(1-\alpha)(4 - \alpha(15 - 7\alpha)) - 3\alpha(8 - (3-\alpha)\alpha)]),\\
C_2(\cdot) &= (1-\alpha)\left[16(1-k)^3k^2 - 48\alpha + 16k(7 + k[-11 + k(16 + k(-13 + 4k))])\alpha\right.\\
&\qquad + 4(1-k)[13 + 2k(-3 + 2k)(5 + 6(-1 + k)k)]\alpha^2\\
&\qquad \left. - 4(3 - 4k)(1 - 3k + 2k^2)^2\alpha^3 + (1-k)(3 - 2k)(1 - 2k)^3\alpha^4\right],
\end{aligned}$$

and

$$\begin{aligned}
Y(p_1, k, \alpha, c) &= D(\alpha, c) \times E(p_1, k, \alpha, c) \times F(p_1, k, \alpha, c),\\
D(\cdot) &= 4c\alpha,\\
E(\cdot) &= -3(1-k)(1-\alpha)p_1 + 2c\alpha[3 - 2k(1-\alpha) - \alpha]\alpha,\\
F(\cdot) &= [2c\alpha - p_1(1-k)(1-\alpha)]^3.
\end{aligned}$$

We are exploring a region where $c > 0$, $\frac{2k+\sqrt{3+2k+4k^2}}{2(3+2k)} < \alpha < 1$, $0 \le k < 1$, $p_1 \in \left(c\alpha, \frac{4c\alpha}{3}\right)$. In this parameter region, it can be shown that $D(\cdot) > 0$, $E(\cdot) > 0$, $F(\cdot) > 0$. Thus $Y(p_1^*(k), k, \alpha, c) > 0$. Thus, the sign of $\frac{d\pi_S}{dk}$ is given by the sign of $X(\cdot)$. Since $A(\cdot) < 0$, the sign of $X(\cdot)$ is opposite to the sign of $B(\cdot)$. Equation $B(p_1, \cdot) = 0$ is quadratic in p_1 and has two real solutions $\varrho_1(k, \alpha, c) \le \varrho_2(k, \alpha, c)$. It can be shown that $\varrho_2 > \frac{3c\alpha}{2} > p_1^*(k)$. Moreover, $\lambda_{c,\alpha}(\varrho_1, k) > 0$. Thus, from the ending argument in the proof of Lemma E1 we see that it must be the case that $p_1^*(k) > \varrho_1$. Thus $p_1^*(k) \in (\varrho_1, \varrho_2)$. Moreover, it can be shown that in the considered parameter region we have $C_2(\cdot) < 0$. Consequently, $B(p_1^*(k), \cdot) > 0$. Thus $X(p_1^*(k), k, \alpha, c) < 0$.

Consequently, $\frac{d\pi_S}{dk} < 0$ in this region of the parameter space. Consequently, the firm wants to decrease seeding ratio. Case (i) under positive seeding ratio cannot be optimal. When $\frac{1}{2} \le \alpha < 1$, we obtain $k_S^* = 0$. When $\frac{1}{2\sqrt{3}} < \alpha < \frac{1}{2}$, we are pushed into case (ii) as decreasing k makes it reach lower bound for case (i.b).

(ii) $\frac{1}{2\sqrt{3}} < \alpha < \frac{1}{2}$ and $0 \le k < \frac{12\alpha^2-1}{8\alpha(1-\alpha)}$. Then, we have $\frac{1}{2\sqrt{3}} < \alpha \le \frac{2k+\sqrt{3+2k+4k^2}}{2(3+2k)} < \alpha < 1$. For a given seeding ratio k, we are in case (a) of Lemma E1. In this case, we have a closed form expression for $p_1^*(k)$ and $\pi_S^*(k)$ and we can show again that $\frac{d\pi_S}{dk} < 0$. Details are omitted for brevity. Thus, the firm has no incentive to seed. Thus $k_S^* = 0$.

Case 3. $0 < \alpha \le \frac{1}{2\sqrt{3}}$. Note that for any $k \ge 0$ we have $\frac{1}{2\sqrt{3}} \le \frac{2k+\sqrt{3+2k+4k^2}}{2(3+2k)}$. Thus, under both S and CE we have the same solution as in the case of no price commitment and myopic customers. In this region, the solution is the same as the one in Proposition D2. □