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Competitive Options, Supply Contracting, and Electronic Markets

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This paper develops a framework for analyzing business-to-business (B2B) transactions and supply chain management based on integrating contract procurement markets with spot markets using capacity options and forwards. The framework is motivated by the emergence of B2B exchanges in several industrial sectors to facilitate such integrated contract and spot procurement. In the framework developed, a buyer and multiple sellers may either contract for delivery in advance (the "contracting" option) or they may buy and sell some or all of their input/output in a spot market. Contract pricing involves both a reservation fee per unit of capacity and an execution fee per unit of output if capacity is called. The key question addressed is the structure of the optimal portfolios of contracting and spot market transactions for the buyer and these sellers, and the pricing thereof in market equilibrium. Existence and structure of market equilibria are characterized for the associated competitive game between sellers with heterogeneous technologies, under the assumption that they know the buyer's demand function. This allows an explicit characterization of the price of capacity options and the value of managerial flexibility, as well as providing conditions under which B2B exchanges are efficient and sustainable.

Key words: capacity options; B2B exchanges; contracting; flexibility; supply chain management; competitive equilibrium; technology choice
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1. Introduction

This paper studies business-to-business (B2B) exchanges and flexible supply chain contracting that rely on capacity options in a competitive environment. We focus on the case of a single buyer, with extensions to the case of multiple buyers following directly from this. In this scenario, sellers compete to supply the buyer in a market in which, in the short run, capacities and technologies are fixed. The buyer can reserve capacity through options obtained from any seller. Output on the day can be obtained either through executing such options or in a spot market. Such contract spot markets have become more prominent under e-commerce (e.g., Grey et al. 2005, Laseter et al. 2001, Wise and Morrison 2000), and include commodity chemicals, electric power, metals, natural gas, plastics, and semiconductors.

A common feature of electronic markets supporting such capital-intensive industries is the following. Any particular buyer has only a small set of sellers who compete for the buyer's business in the contract market, while still having access to a larger set (sometimes a much larger set) who compete in the shorter-term market (the spot market) and whose actions determine a competitive spot market price.[1] Contract sellers for a particular buyer are restricted to a prequalified set who are able to satisfy credit and settlement requirements, assurance of supply, access to supporting logistics, and other traditional supplier management issues. These features give rise to a setting in which buyers have restricted seller bases (of perhaps one to five sellers) in their contract markets, while using spot markets as a second source of supply as well as a means of evaluating the price levels they receive in their contract purchases. The interaction between contracting and spot market purchases is, thus, of interest both in the optimal portfolio of seller sourcing for a buyer, as well as in providing an interconnected valuation process. We provide

[1] For details on B2B exchanges, and those that have survived the recent shakeout, see Day et al. (2003), who note the importance of prescreening of sellers by contract buyers, using the exchange as an information exchange platform. This prescreening is an important survival condition for assuring continuing participation in B2B exchanges that feature both support of contract sellers, as well as open exchanges with larger numbers of participating "spot firms."

a framework for understanding this setting that is motivated by existing conditions in several capital-intensive industries.

Consider the beverage industry. Aluminum is an extremely important element of the cost structure. For major buyers like Anheuser Busch, a restricted set of sellers is used, even though the aluminum spot market price is a key benchmark for sourcing and hedging and is determined by the actions of scores of global players. First, sourcing arrangements with main sellers are typically set according to the spot price plus processing costs, and contracts are marked to market on a daily basis. Thus, AAA credit rating is essential for the main contract partners. Second, there may be value-added services undertaken by these contractors to take aluminum ingots and prepare them in a more suitable fashion for can production, and here again this would be done only with specifications for these services worked out with a few sellers. Thus, even though from time to time the mix of contract sellers may change, in the short and medium run, only a restricted set of contract sellers is used under current industry practice in commodity metals.[2] Similar findings are in Levi (2002), who provides a detailed empirical study of procurement decisions in the chemical industry. Levi (2002) demonstrates the importance of contract relationships, typically among one to five sellers, as well as the importance of short-term (effectively, spot) markets for goods that are commodities.

In the newly restructured electricity market (e.g., Wilson 2002), producing sellers (generators) and buyers (load-serving entities and distribution companies) can sign bilateral contracts to cover the demands of their retail and wholesale customers. These bilateral contracts may cover purchases for up to a year in advance. Alternatively, sellers and buyers can interact "on the day" in a spot market. How much of their respective capacity and demand sellers and buyers should or will contract for in the bilateral contracting market, and how much they will leave open for spot transactions, is a fundamental question examined in a growing literature on energy trading (e.g., Clewlow and Strickland 2000, Kleindorfer and Li 2005). Moreover, capacity options of the sort analyzed in this paper are under consideration for operating reserves and ancillary services as well as in wholesale energy markets.[3]

The same general market structure obtains in the semiconductor industry (e.g., Cohen et al. 2003), at least for the most popular memory chip device families. Even though memory chips such as DRAMs (dynamic random access memory) are commodities, major manufacturers source over 90% of their components only from a restricted set of sellers, while fulfilling their residual demand from the spot market. The current trend of "structured sourcing" in the semiconductor industry is to explore options-based contracts, which could effectively hedge price risks, supply risks, and reduce cost via better coordination with a buyer's prequalified and restricted seller base.[4]

The key question we address, which is unresolved in previous literature, is the consequence of competition among multiple sellers with heterogeneous technologies. This question can be addressed in two ways: one is to assume a closed spot market, in which all participants in the contract market also participate in the spot market, and vice versa; alternatively, one can assume an open structure in which the spot market price is determined by a larger group of competitive sellers than the smaller, restricted group operating in the contract market. In this paper, we follow the latter assumption, as this better fits the applications we have in mind in capital-intensive industries. This paper characterizes the structure of efficient contracts in this setting and the conditions for the existence of an integrated contract spot market equilibrium among competing sellers who are equipped with heterogeneous technologies. This allows an explicit characterization of the price of capacity options and the value of managerial flexibility, as well as providing conditions under which B2B exchanges are efficient and sustainable as market-clearing platforms for contracting and spot purchases.

The rest of this paper is organized as follows. Section 2 provides a brief literature review. Section 3 sets up our model. Section 4 derives optimal strategies for the buyer and the sellers and characterizes the necessary and sufficient conditions for the existence of the equilibrium. Section 5 analyzes efficiency of equilibrium contracts. Section 6 presents some extensions based on our framework, including the analysis of imperfect substitution of contract spot purchase. Section 7 concludes with summary and discussion.

2. Literature Review

Several streams of previous work relate to ours. We now briefly review them and show that our framework generalizes several key results in the literature.

Any discussion of options contracts must begin with the foundations established in the financial economics literature, e.g., Dixit and Pindyck (1994), Kawai (1983), Newbery and Stiglitz (1981), and Trigeorgis (1996). However, this stream of work neglects the interaction

[2] Private communication: Steven R. Iverson, Executive Director, Metals and Energy Procurement, Anheuser Busch Company.

[3] See, e.g., 5.1.14 of CAISO (2002).

[4] Private communication: Dr. Thomas Olavson, Manager of Supply Chain Operations, Hewlett Packard; Jim Feldhan, President, Semico Research Corporation.

of fixed capacities in the short run, and the heterogeneity of costs (so-called "diverse technology") across competing sellers. However, these operational issues are rather fundamental to most real problems. In economics, work on diverse technologies, e.g., Crew and Kleindorfer (1976), characterizes efficient production decisions and technology mixes under competition and price uncertainty, but this work does not consider contract and spot market interactions. In industrial organization, previous work on forward-spot integration, e.g., Allaz and Vila (1993) neglects either capacity constraints, spot price uncertainty, options, or heterogeneous technologies. Thus, these previous contributions from various fields are all incomplete in terms of modeling the interaction between capacity, contracting, and output decisions.

It appears that Wu et al. (2002) is the first publication in the stream of literature on integrating contract spot purchases. They study the case of a single seller with one or more buyers. A number of significant extensions have since appeared. We briefly review some of this work; for a more complete review, the reader is referred to the survey by Kleindorfer and Wu (2003).

Spinler et al. (2003) generalize the Wu et al. (2002) framework to the state-dependent case, whereby the willingness-to-pay (WTP) functions characterizing demand for buyers could themselves depend on the state of the world (e.g., both demand and spot price might depend on temperature), showing that the basic structure of Wu et al. (2002) can be extended to stochastic costs and demand.

Mendelson and Tunca (2004a, b) analyze the impact of a closed B2B spot market (the exchange) on the participants as a function of information quality. They show for a single-seller model (with numerical results for the multiseller model) that the introduction of the exchange does not necessarily benefit the participants. Deng and Yano (2003) and Lee and Whang (2002) follow this tradition in examining the role of integrated contract spot purchases in a two-echelon supply chain in the setting of a closed spot market, showing net benefits for the buyer but not necessarily for the seller.

Cheng et al. (2003), Deng and Yano (2002), as well as Golovachkina and Bradley (2002, 2003) generalize the Wu et al. (2002) model to study whether options-based contracts can coordinate the entire supply chain. The buyer in their model is a manufacturer that purchases from the seller and sells products for profit. As a result of the differences in their underlying model (the buyer has only a single seller to source from, both in the contract and the spot market), they show numerically that the two-part options contract may not always be efficient for one-on-one contracting, i.e., in the absence of competition. Anand et al. (2003) show that options contracts may not coordinate the supply chain if the seller can utilize inventory strategically. This contrasts with the results below, where optimal options contracts under competition are shown to be efficient. Peleg et al. (2002) study minimum-take contracts, where they consider three different sourcing strategies: contract with one seller, online search (open spot market), and a combination of the two. They show conditions under which one is better than the other, including when a combined contract/spot strategy is efficient.

The supply chain contracting literature in operations management (OM) has also considered problems similar to those studied here, but lacking several components: competition, heterogeneous technologies, and the integration of these with B2B exchanges. The prices and contract terms in most of the OM contracting literature are predetermined through a negotiation framework rather than through a competitive, market-equilibrium approach as pursued here. Furthermore, demand uncertainty in these papers is assumed to be exogenous and not linked to spot market price uncertainty. These models typically assume that excess demand may be lost, possibly with additional penalties, and excess supply may be sold in a "salvage market" (which may be thought of as similar to a spot market, but at a fixed price). A basic review of these results can be found in Cachon (2003), Kleindorfer and Wu (2003), Swaminathan and Tayur (2003). Linking this literature with the options framework here, several papers (Araman et al. 2002, Barnes-Schuster et al. 2002, Shi et al. 2002) show that buy-back contracts, quantity-flexible contracts, information-sharing contracts, as well the classic newsvendor model and others are all special cases of two-part-tariff options (however, these papers only treat the single-seller, single-buyer case).

The closest research to ours is Perakis and Zaretsky (2004) and Martínez-de-Albéniz and Simchi-Levi (2003b). This work generalizes the Wu et al. (2002) model to include seller competition but makes an additional, crucial assumption that capacity is immediately scalable at the time contracts are signed. They derive similar results to our basic case and provide a number of extensions, including the multiperiod case. In our model, capacity is assumed to be fixed and not variable in the short term, which is a central feature of the capital-intensive industries of interest here.

This paper builds on our earlier work, but the extension to multiple sellers leads to significant new contributions and insights. Specifically, the model combines several components (forwards, options, diverse technology, and contract spot market integration) that have been dealt with separately in the literature of finance, economics, and supply chain management. The results obtained include the equilibrium price and market structure/segmentation, as well as several other contributions on the factors that underlie the efficient use of options contracts.

3. A Description of the Model

In this section, we present our model.

There are I sellers and only one aggregate buyer to transact via a B2B exchange for a good.[5] Let $\Xi \stackrel{\text{def}}{=} \{i\colon i = 1, \ldots, I\}$ be the set of sellers[6] who are available to participate both in the contract and the spot market (of course, there may be many sellers who are outside of this set and who only participate in the spot market). The heterogeneous technologies of the sellers are characterized by (b, K), where b is the seller's short-run marginal cost of providing a unit, and K is the seller's total available capacity. In the short run, these technologies are assumed to be fixed.

The timing of decisions is captured in a two-period framework. Seller and the buyer sign option contracts in advance (Period 1), and then "on the day" (Period 2), after the spot market price is revealed, they decide how much to deliver/exercise from the contract and how much to sell/purchase on the spot market. In reality, options contracts typically have a maturation date before, but rather close to, the actual spot market, e.g., a week or a day in advance of the spot market. For example, in electric power, options may be called up to 24 hours in advance of delivery. Earlier execution dates are typical in calling capacity options in semiconductors. Our assumption that the spot price is known at the time options are exercised is therefore an approximation that fits some settings better than others.

The decision variables to the sellers are the optimal contract $[s, g, L]$ to offer to the buyer, where s is the reservation cost per unit of capacity, g is execution cost per unit of output, and $L \leq K$ is the seller's capacity bid to the contract market. We assume throughout that sellers face stringent penalties for nonperformance under contract so that they will, in fact, bid no more than $L \leq K$ as available capacity and that they will set prices (s, g) so that contracted amounts will not exceed L. Note that the structure of the two-part contract (s, g) is standard in the options literature as well as in practice.[7]

The decision variables to the buyer are how much to contract Q in advance with the sellers, and on the day, how much to execute from the contract $q \leq Q$ and how much to procure from the spot market x. The buyer's total demand on the day (at Period 2) $\sum_{i=1}^{I} q_i + x$ is assumed to be common knowledge, where P_s is spot market price.

[5] Generalizing the analysis to multiple buyers is straightforward, i.e., by allocating capacity according to the buyer's WTP, as shown in Wu et al. (2002).

[6] We suppress the subscript i for each seller i when it is clear.

[7] We thank Andy Huemmler of Exelon Corporation and Robert Levin of the New York Mercantile Exchange for valuable discussions regarding existing options markets, where such contracts are routinely used.

The spot market price P_s is assumed to be exogenous, i.e., the spot market is open and none of the sellers in or out of the set Ξ has any market power in influencing it. The open spot market is a key assumption of our model, which captures the essential common feature of several electronic markets we described earlier. This is a reasonable approximation when there are a sufficient number of participants in the spot market, whether or not they participate in the contract market.

The objective of the buyer is to maximize a given utility function (as specified below) subject to available option contracts. The objective of the sellers is to maximize expected profit jointly obtained from both the contract market and the spot market, taking into consideration the buyer's reaction and competition from other sellers.

We assume both the sellers and the buyer are risk neutral. If sellers were risk averse, they would naturally be more active in the contract market, using it as a hedge against low prices in the spot market. Even when they are risk neutral, a key factor influencing their incentives to sign contacts is imperfect market access on the day, reflecting illiquidity (Mendelson and Tunca 2004b) of the spot market or access constraints to it. To capture this "access risk," we define $m(P_s)$ as the probability that sellers can find a last-minute buyer on the spot market when the realized spot price is P_s. The lack of liquidity or imperfectness of spot market access has been widely cited as the killer for public B2B exchanges (e.g., Day et al. 2003). We will further justify and discuss this assumption after we have derived our main results. In a nutshell, we are assuming that there is some risk, e.g., a price loss of $(1 - m(P_s))$ per unit of sale, to relying on the spot market as a second source. The risk is reflected here in terms of access to this second source, but additional risk elements for second sourcing are standard in the supply chain contracting literature. For example, such markets have been treated as "salvage" markets for the seller's excess supply where sellers incur losses in margin from spot sales (e.g., Lee and Whang 2002; Golovachkina and Bradley 2002, 2003), or as the additional cost of last-minute orders if they use this market to sell excess (uncontracted) capacity. The net effect of this margin loss is equivalent to the effect of less than perfect access assumed here. Indeed, one may simply think of $(1 - m(P_s) > 0)$ as the fraction of margin lost through sales to the spot market rather than to the more orderly contract market.

The game played is a one-shot, static, pure-strategy Bertrand-Nash game among sellers making simultaneous bids on contract-offering strategies (s, g, L). The payoff of each seller is its own expected profit, taking into account the total demand function of the buyer. Thus, each seller is a von Stackelberg leader.

This is consistent with sophisticated sellers and well-developed markets. We assume that sellers' technologies are private information. Sellers bid simultaneously their competitive contract offer to the buyer. We refer to this equilibrium, when it exists, as a Bertrand-Nash Equilibrium (Fudenberg and Tirole 1991).

We state our assumptions below and refer the reader to the appendix for a summary of key notation.

To begin with, we assume the buyer's underlying preferences can be represented in the quasi-linear form: $V(z, n) = U(z) + n$. The first term $U(z)$ is the buyer's willingness to pay and satisfies the assumption below; the second term n can be thought of as "money."

Assumption 1 (A1). *The buyer's WTP $U(z)$ is strictly concave and increasing so that*

$$U'(z) > 0, \ U''(z) < 0, \quad \text{for } z \geq 0. \tag{1}$$

Both the quasi-linear form of $V(\cdot)$ and A1 are standard assumptions in economics (Mas-Colell et al. 1995).

Denote $D_s(v)$ as the buyer's normal demand function when there exists only the spot market, i.e., $D_s(v) = \arg\max_{D\geq 0}\{U(D) - vD\}$, so that $D_s(v) = (U')^{-1}(v)$. From A1, it follows that $D_s(v)$ is monotonically decreasing.

Further, we assume some rational choice behavior of the buyer when contracting capacity via options.

Assumption 2 (A2). *No-Excess Capacity Condition: Let sellers' offers be indexed so that $g_1 \leq g_2 \leq \cdots \leq g_I$. Then the No-Excess Capacity Condition is said to hold if and only if*

$$Q_i\left[D_s(g_i) - \sum_{l=1}^{i} Q_l\right] \geq 0, \quad i = 1, \ldots, I.$$

A2 implies that the buyer will not contract for any capacity she is sure she will not use on the day. This condition says that if $Q_i > 0$, then the sum of all contracted capacity $\sum_{l=1}^{i} Q_l$ with execution fees less than or equal to g_i must not exceed $D_s(g_i)$. This condition is without loss of generality for any rational buyer because, on the day, the optimal execution order of contracts is to use all contracts available in order of increasing g_i up to the point at which the next such execution fee exceeds P_s, satisfying all additional demand from the spot market. Thus, if A2 were violated, it would mean that the buyer had contracted for some capacity that would be guaranteed never to be used on the day. This follows because the buyer would have to be willing to pay at least the execution fee g_i per unit of output in order to make use of contract capacity, and $D_s(g_i)$ captures all demand for which the buyer is willing to pay at most g_i to cover on the day. This nonoptimal behavior is ruled out by the condition stated. We state this as an assumption here both to clarify the nature of the buyer's choice and to rule out certain knife-edge cases in the proofs that are uninteresting in equilibrium (e.g., the condition simplifies the consideration of cases in which there are ties in the bid g_i; see also Remark 2 in §4.3.).[8]

Finally, we specify an allocation rule in case there is a bid tie in the seller's contract bids.

Assumption 3 (A3). *Bid-Tie Allocation Mechanism: When there is a bid tie among a set of sellers $M \subseteq \Xi$, then the buyer's demand for seller i's output is proportionally allocated to the sellers according to their bid capacity, thus $Q_i = D(p)K_i / \sum_{i\in M} K_i$.*

This proportional bid-tie allocation rule is often used in practice; it would also be the expected outcome of a process in which random allocation among sellers with the same bid occurs. This type of allocation mechanism is typical in the literature on price competition (e.g., Friedman 1988). The results given on the demand side below are not very sensitive to the manner in which bid ties are broken. However, the equilibrium results for the sellers below can be affected by varying assumptions, demonstrating the importance of governance rules for B2B exchanges. For further analysis on allocation rules, see Cachon and Zhang (2005) and Kremer and Nyborg (2004).

We note here that all proofs can be found in the e-companion website of *Management Science* (http://mansci.pubs.informs.org/ecompanion.html), unless otherwise noted as trivial.

4. Optimal Strategies for the Buyer and Multiple Sellers

In this section, we first derive the buyer's optimal strategies, then the sellers' optimal strategies, necessary and sufficient conditions for the existence of the Bertrand-Nash equilibrium among sellers, uniqueness and computation of such an equilibrium when it exists, and finally, the price of real options and the value of flexibility in supply chain contracting.

4.1. The Buyer's Problem

Define the buyer's utility as follows.[9]

$$V(q, x; P_s, Q) = U\left(\sum_{i=1}^{I} q_i + x\right) - \sum_{i=1}^{I} s_i Q_i - \sum_{i=1}^{I} g_i q_i - P_s x, \tag{2}$$

[8] Similar conditions are either implicitly assumed or derived by Perakis and Marina (2004), Martínez-de-Albéniz and Simchi-Levi (2003b).

[9] Note that this does not imply anything about risk preferences. The buyer is, in fact, risk neutral in our model because $V(\cdot)$ is linear in money.

where q is the vector of purchases under contract from the sellers, and x is the amount purchased from the spot market.

The buyer solves her sourcing problem via a two-step backward induction. At Period 2, she derives her optimal consumption (q, x) given the realization of spot price P_s and her contracted capacity Q on hand. Working backward, at Period 1, she decides how much to contract Q with the sellers at Period 1.

We use Lemma 1 and Theorem 1 to present the buyer's optimal strategies at each period.

LEMMA 1. *Let (P_s, Q) be given. Without loss of generality, assume that sellers' offers are indexed so that $g_1 \leq g_2 \leq \cdots \leq g_I$. Let the buyer's optimal consumption be given by the solution (q, x) to the following problem:*

$$\operatorname*{Max}_{q,x} V(q, x; P_s, Q), \tag{3}$$

subject to

$$x \geq 0, \qquad Q \geq q \geq 0. \tag{4}$$

Then the solution (q, x) to (3)–(4) gives optimal purchases under contract from seller $i = 1, \ldots, I$ and from the spot market:

$$q_i(P_s, Q) = Q_i \chi(P_s - g_i),$$
$$x(P_s, Q) = \left(D_s(P_s) - \sum_{l=1}^{k} Q_l\right)^+, \tag{5}$$

where $\chi(\cdot)$ is the indicator function (which takes the value of 1 if its argument is positive and 0 else), seller $k = k(P_s, Q)$ provides the last unit of contract output to the buyer.

Defining $g_0 = 0$ and $g_{I+1} = \infty$, seller k is determined as the first seller (in the indicated order of increasing g_i) satisfying $g_k < P_s \leq g_{k+1}$. If $k = 0$, no contract capacity whatsoever is used.

Define $p \stackrel{\text{def}}{=} s + G(g)$, with the "effective price function" $G(v)$ defined as $G(v) \stackrel{\text{def}}{=} \mathrm{E}\{\min(P_s, v)\}$, where G^{-1} is the inverse function of G. Define $D(v) \stackrel{\text{def}}{=} D_s(G^{-1}(v)) = (U')^{-1}(G^{-1}(v))$.

Define $X(M) = \sum_{i \in M} L_i$ as the total bid capacity of all sellers in set M. For every $k \in M \subseteq \Xi$, we define the following four sets, $M_k^{1,2,3,4}(s, g) = \{i \in \Xi \mid p_i <, \leq, =, > p_k\}$, depending on the comparison of p_i and p_k respectively.[10]

THEOREM 1 (BUYER'S OPTIMAL CONTRACT STRATEGY). *Given Assumptions* 1, 2, *and* 3 *and sellers' bid (s, g, L); without loss of generality, assume that seller bids are ranked in order of the index $p_i = s_i + G(g_i)$. If $G(U'(0)) \leq \min\{p_i\}$, then the buyer's solution will be to set $Q_i = 0, \forall i$, i.e., no contracting is optimal. Otherwise,* Greedy Contracting *in order of p_i is optimal for the buyer, i.e., the optimal portfolio of contracts has the form $\forall i \in M_h^1$, $Q_i = L_i$; $\forall i \in M_h^4$, $Q_i(s, g, L) = 0$; and for $i \in M_h^3$*

$$Q_i(s, g, L) = \frac{L_i}{X(M_h^3)}\big(D(p_i) - X(M_h^1)\big), \tag{6}$$

where $h \in \{1, \ldots, I\}$ is any seller (there may be more than one in the case of tied bids) with the largest value of the index p_i satisfying

$$p_h < G\big(U'(X(M_h^1))\big). \tag{7}$$

The structure of the optimal portfolio captured in Theorem 1 is relatively simple.[11] It calls for the buyer to rank all offers in terms of a single index p_i and then to pull off as much capacity as allowed by seller i, proceeding in rank order of the contract index until the marginal WTP is exceeded by the contract index. Because $p_i = s_i + G(g_i)$ and G (and therefore G^{-1}) is strictly increasing ($G'(x) = 1 - F(x) > 0$), and since by concavity $D_s(\cdot)$ is (strictly) decreasing, therefore Q_i is decreasing in p_i for all $i \in \Xi$. Of course, WTP may be exceeded with the first seller and the buyer may, in fact, sign no contracts whatsoever (if $G(U'(0)) \leq p_1$). Note that once contracts are signed, they are executed on the day by the buyer in order of increasing g_i rather than p_i.

4.2. Sellers' Problem and the Bertrand-Nash Equilibrium

In what follows, we first present sellers' optimal capacity bidding strategy (L^*), then the optimal bidding for the option execution fee (g^*), and finally the optimal pricing of the option in the associated market equilibrium (s^*).

The profit of seller i is given by

$$\pi_i(s_i, g_i, L_i, P_s) = s_i Q_i + (g_i - b_i) q_i + (P_s - b_i)^+ m_i(P_s)(K_i - q_i),$$

where, from Lemma 1, $q_i = Q_i \chi(P_s - g_i)$. Expected profit is therefore given by

$$\begin{aligned} \mathrm{E}\pi_i(s_i, g_i, L_i, P_s) &= s_i Q_i + (g_i - b_i)(1 - F(g_i)) Q_i \\ &\quad + (K_i - Q_i)\int_{b_i}^{\infty} (P_s - b_i) m_i(P_s)\, dF(P_s) \\ &\quad + Q_i \chi(g_i - b_i) \int_{b_i}^{g_i} (P_s - b_i) m_i(P_s)\, dF(P_s). \end{aligned} \tag{8}$$

Seller i's problem is to choose (s_i, g_i, L_i) so as to maximize its expected profit $\mathrm{E}\pi_i(\cdot)$ from both the contract market and the spot market, subject to the constraint that $Q_i(s, g) \leq L_i \leq K_i$, where seller i assumes

[10] However, we will usually suppress the dependence of the sets $M_k^{1,2,3,4}$ on (s, g) in the theorem.

[11] Theorem 1 includes the Wu et al. (2002) single-seller result, $Q = D(p_1) = D(s_1 + G(g_1))$ as a special case.

that other sellers' prices (s_{-i}, g_{-i}) are fixed. However, note that the expected profit function $E\pi_i(\cdot)$ does not depend directly on L_i. Thus, if seller i bids in the contract market at all, relaxing the feasible set to the maximum available capacity by setting $L_i = K_i$ cannot decrease profits.[12]

The following lemmas are straightforward extensions of the Wu et al. (2002) results to the case of multiple sellers.[13]

Lemma 2 (Sellers' Optimal Capacity Bids). *$L_i^*(L_i^* - K_i) = 0, \forall i$, so that any seller who posts nonzero capacity in the contract market will post all capacity, $L_i^* = K_i$.*

Lemma 3 (Sellers' Optimal Execution Fee Bids). *$g_i^* = b_i, \forall i$.*

Define $c_i \stackrel{\text{def}}{=} \underline{s_i} + G(b_i)$, where $\underline{s_i} \stackrel{\text{def}}{=} E\{m_i(P_s)(P_s - b_i)^+\}$ is the seller's unit opportunity cost on the spot market if the buyer chooses to exercise her contract. Substituting $g_i^* = b_i$ in (8), we have the following profit functions:

$$E\pi_i(s, b, K, P_s) = [s_i - \underline{s_i}]Q_i(s, b, K) + \underline{s_i}K_i;$$

with $p_i = s_i + G(b_i)$, this can be rewritten as

$$E\pi_i(p) = [p_i - c_i]Q_i(p) + [c_i - G(b_i)]K_i, \qquad (9)$$

where $Q_i(p) = Q_i(s, b, K)$ is given by Theorem 1. Because the second term $[c_i - G(b_i)]K_i$ is fixed with regard to the decision variables, we will suppress it in the following analysis. We are interested in the optimal price of the options s_i of the sellers, but we will characterize equilibrium of interest in p-vector space because $s_i = p_i - G(b_i)$. The following lemma states that, this equilibrium price vector p, if exists, can be only one price for all participating sellers in the contract market.

Lemma 4. *If there exists an equilibrium, then it must be symmetric for all sellers providing positive capacity in the contract market. That is, every equilibrium p must be of the form $p_i = p_j, \forall i, \forall j \in M$, where $Q_i(p) > 0$, $i \in M$ and $Q_k(p) = 0$, $k \in \Xi \backslash M$.*

An immediate consequence of the "law of one price" of Lemma 4 is the "law of full-capacity contracting," which says that if an equilibrium exists with two or more sellers active in the equilibrium contract market, then the entire capacity of every seller in the equilibrium will be contracted by the buyer. Otherwise, the partial capacity-contracted seller (say seller i) will have an incentive to reduce the price a little bit by setting $p^{\text{new}} = p - \epsilon$, where p is the equilibrium price and ϵ is any value that is smaller than $p(1 - Q_i/K_i)$. Thus, seller i would be able to get its entire capacity contracted, keeping all other players' strategies fixed at their equilibrium values. However, this is a direct violation of the above "law of one price" and a contradiction to the equilibrium conditions. Put differently, although it might be optimal for the buyer to contract a fraction of the seller's capacity, as indicated by Theorem 1, this case is ruled out in a competitive equilibrium with two or more sellers. This contrasts with previous literature on the single-seller case, i.e., no competition among sellers. In the single-seller case, partial contracting $(Q < K_1)$ can be optimal (e.g., Cheng et al. 2003; Golovachkina and Bradley 2002, 2003; Spinler et al. 2003; Wu et al. 2002).

Given Lemma 4, we denote by p the equilibrium price, such that for those sellers $i \in M$ (where M is the short-term equilibrium set) having positive contract capacity at equilibrium, $p_i = p$, $\forall i \in M$. Denote $|M|$ as the number of sellers in set M.

We note the following fact, which is needed in the Theorem below. If seller k participates in the equilibrium and bids p_k as a final unit provider in Theorem 1, then (assuming all other bidders keep their bids constant) seller k's own profit function for increases in price p_k is given by $E\pi_k(p_k) = [p_k - c_k]Q_k(p_k) = [p_k - c_k][D(p_k) - \sum_{i=1}^{k-1} K_i]$. Assuming pseudoconcavity of $[p_k - c_k]D(p_k)$, as in standard economics (e.g., Friedman 1988), then $E\pi_k(p_k)$ is also pseudoconcave.[14] Denote D^{-1} as the inverse function of D.

Theorem 2 (Bertrand-Nash Equilibrium). *Let (K, p, M) be any short-term equilibrium, where $M \subseteq \Xi$ is the equilibrium set of all sellers having positive capacity contracts, i.e., $Q_i(p) > 0$, $i \in M$ and $Q_k(p) = 0$, $k \in \Xi \backslash M$. If $\text{Min}\{c_i \mid i \in \Xi\} \geq G(U'(0))$, then no seller will participate in the contract market. Suppose, therefore, that $\text{Min}\{c_i \mid i \in \Xi\} < G(U'(0))$. Then, if M is a single-seller equilibrium set with $|M| = 1$, the equilibrium price is $p = \max\{\arg\max_{p_1}(p_1 - c_1)D(p_1), D^{-1}(K_1)\}$, where $c_1 = \text{Min}\{c_i \mid i \in \Xi\}$* (Wu et al. 2002). *Otherwise, if the equilibrium set M consists of two or more sellers with $|M| > 1$, then the necessary and sufficient conditions for such a nonsingleton equilibrium p to exist are*

C1. $p = D^{-1}(\sum_{k \in M} K_k)$
C2. $\partial E\pi_k(p_k)/\partial p_k \leq 0$, *if* $p_k > p$, $\forall k \in M$
C3. $\forall k \in \Xi \backslash M$, $p < c_k$.

Condition C1 says that if any equilibrium exists, it must be symmetric for all sellers in the equilibrium, regardless of their diverse technologies. Specifically, they enjoy the same contract price, and every

[12] Note that this is only the seller's optimal capacity bid; it does not imply that the seller will always sell its whole capacity to the buyer. Rather, the actual capacity transacted is $Q_i(s, g)$, which depends on his two-part tariff (s, g) and cannot exceed his bid capacity $(Q_i \leq L_i \leq K_i)$.

[13] For proof of Lemma 2, see preceding text. The proof of Lemma 3 is omitted, as it is analogous to that of Wu et al. (2002).

[14] As shown below, in the special case where all sellers are identical, this function becomes $E\pi_k(p_k) = [p_k - c_k]Q_k(p_k) = [p_k - c_k][D(p_k) - (I - 1)K_k]$.

seller's entire capacity will be reserved. We will show below that in the special case of identical sellers, the contract market has to support all participants, with only one contract price as $p = D^{-1}(IK_1)$. The intuition for the other two conditions is clear—they are Nash equilibrium conditions. Condition C2 ensures that no sellers in the current equilibrium set M would have any incentive to unilaterally deviate from the current contract price. Condition C3 implies that no sellers outside of the current equilibrium set M have any incentive to join the equilibrium bidders by bidding a price that is equal to or lower than the current contract price p, as doing so results a net loss in its profit.

If any of the above three conditions (C1, C2, C3) is violated, then no equilibrium is the outcome. In this case, the model suggests "cycling" behavior—some sellers may frequently enter and exit the contract market. We hypothesize seller's technology index c_k would be a key driving factor that causes such cycling behavior, which is frequently observed in practice in industries with scale economies, such as the semiconductor sector (see, e.g., Cohen et al. 2003, Semico Tracker Newsletter 2003) and the electric power sector (see, e.g., Bunn and Day 2003, Wilson 2002).

Corollary 1 (Identical Sellers). *In the case of identical sellers, if an equilibrium exists, then it entails all I sellers participating in the contract market,* $M = \Xi$, *and the contract price is* $p = D^{-1}(IK_1)$.

Theorem 3 (Uniqueness and Computation of Equilibrium). *If there exists any equilibrium* (K, p, M) *for some* $M \subseteq \Xi$, *then it must be unique. Furthermore, if such an equilibrium exists, it can be computed as follows: Index sellers in order of* c_i, *i.e.,* $c_1 \le c_2 \le \cdots \le c_I$, *and, for convenience, define* $c_{I+1} = \infty$. *Let* $M = \{1, \ldots, h-1\}$, *where h is the smallest index that satisfies* $G(U'(\sum_{i=1}^{h-1} K_i)) < c_h$. *If* C2 *and* C3 *are both satisfied for the corresponding equilibrium price* $p = D^{-1}(X(M))$ *determined by* C1 *of Theorem* 2, *then* (K, p, M) *is the unique equilibrium; otherwise, no equilibrium exists.*

4.3. The Price of Real Options and Value of Flexibility

We now discuss the pricing of the real options considered here and the value of flexibility.

The price of each option by the seller is determined by $s = p - G(b) = D^{-1}(X(M)) - G(b)$, which clearly depends on the seller's technology in competition with other sellers (in the equilibrium set M), the spot market distribution, as well as the buyer's WTP. It is quite intuitive to see that sellers who have superior technology can price their options higher, and vice versa.

The value of the flexible supply chain options contract can be explicitly calculated as $s - \underline{s} = p - G(b) - \underline{s}$. To illustrate, assume m is fixed and does not depend on P_s. Then we obtain $s - \underline{s} = p - m\mu - (1-m)G(b)$. From this, it is clear that the value of flexibility in the seller's supply chain contract depends on the mean of the spot market price, market access conditions, as well as the seller's technology. The value moves counter to each of these factors, which is quite intuitive.[15] In any case, a positive value of $p - m\mu - (1-m)G(b)$ is the seller's condition to participate in the contract market. We summarize this in the following remarks.

Remark 1 (Valuable Supply Contracts). Assume fixed m. The conditions for any options contract to have both a nonnegative value to any seller and the buyer are the following:

$$s \ge \underline{s} \text{ or } p \ge c; \quad \text{and} \quad p \le \mu \le G(b) + \frac{p - G(b)}{m}.$$

The second condition gives rise to a threshold value $(p - G(b))/(\mu - G(b))$, for which $m \le (p - G(b))/(\mu - G(b)) \le 1$ must be satisfied in order for the seller to participate in the contract market. This threshold can be thought of as an indicator of how "harsh" it is for the buyer to transact on the spot market (with "unfamiliar" sellers) rather than on the contract market (with "preferred" sellers). It captures the "depth" or "illiquidity" or "thinness" of the spot market. Further implications of this threshold in the presence of other factors will be discussed below.

Remark 2 (No Capacity Arbitrage). None of the sellers or the buyer would contract more than needed and then sell short in the spot market for profit. The reason is that doing so would result in a net margin loss of $m\mu - p \le -(1-m)G(b) \le 0$. The first inequality holds due to the second condition in Remark 1. Hence, selling short is unprofitable. Also, as the law of one price prevails in equilibrium (Lemma 4), arbitrage among sellers or buyers is unprofitable at equilibrium.

Remark 3 (Separating Equilibrium). The equilibrium in Theorem 2, when it exists, defines a short-term market segmentation of sellers as follows: (i) $\forall k \in M$, k participates in the contract market; otherwise, (ii) $\forall k \in \Xi \backslash M$, k will only participate in the spot market.[16]

5. Efficiency of Equilibrium Contracts

In this section, we show that the equilibrium contracts characterized in Theorems 2 and 3 are efficient

[15] For example, for any seller who enjoys perfect market access on the day, i.e., $m = 1$, this value equals $p - \mu$. As the buyer can always wait until the spot price is revealed, $p < \mu$ must be satisfied for any sellable options. If $m = 1$, this gives a strictly negative value and no contract results.

[16] Referred to as "cluster competition," Martínez-de-Albéniz and Simchi-Levi (2003b) show that similar results hold in the absence of a spot market, but under the condition that capacity is immediately scalable.

in the usual sense that they maximize the total surplus for the buyer and sellers engaged in the contract market.[17]

Let $M \subseteq \Xi$ be the fixed equilibrium set characterized in Theorems 2–3. $\pi^o(q(P_s), x(P_s), P_s)$ denotes the joint utility/profit of the buyer and the set of M sellers who have contracted positively with the buyer when the realized spot price is P_s, with an expected value denoted as $E\pi^o(\cdot)$. We denote such a centralized set of the $|M|$ sellers and the buyer as set M^+.

Definition (First-Best Problem for M^+). A *first-best solution* or *centralized allocation* for M^+ is any allocation $(q(P_s), x(P_s))$ maximizing the expected value of joint utility/profit $E\{\pi^o(q(P_s), x(P_s), P_s)\}$ for the buyer and sellers in M^+, where for any realized P_s:

$$\begin{aligned}\pi^o(q(P_s), x(P_s), P_s) &= U\left(\sum_{i\in M} q_i(P_s) + x(P_s)\right) - P_s x(P_s) - \sum_{i\in M} b_i q_i(P_s)\\ &\quad + \sum_{i\in M}(P_s - b_i)^+ m_i(P_s)(K_i - q_i(P_s)) \qquad (10)\end{aligned}$$

such that

$$x(P_s) \geq 0, \quad 0 \leq q_i(P_s) \leq K_i, \quad \forall i \in M.$$

As Groves (1979) shows, the relationship between the defined first-best solution and Pareto efficiency is the following. If the buyer and sellers can choose any allocation $(q(P_s), x(P_s))$ that satisfies capacity constraints, and they can make side payments among one another, then as all utility/profit functions are quasilinear, any Pareto-efficient allocation and payment schedule must maximize the joint surplus defined in (10). All Pareto-efficient outcomes are then determined by varying side payments to achieve prespecified levels of required utility/profit for the M^+ agents involved. We use the term "centralized" here to refer to the fact that if the buyer and sellers comprised a single profit-oriented entity, they would also choose the allocation $(q(P_s), x(P_s))$ maximizing (10).[18]

Theorem 4 (Efficiency of Options Contract for M^+). *Assume $|M| > 1$ so there are two or more sellers. Whenever the equilibrium determined by Theorems 2–3 exists, it is efficient for M^+. In particular, two-part options contracts, properly designed, are efficient in coordinating the supply chain between the $|M|$ sellers and the buyer.*

It is interesting to contrast our findings with previous work. First, consider the debate on efficiency of pure forward contracts versus options contracts. Our two-part options $[s, g]$ are equivalent to forward contracts when $g = 0$; if $g = 0$, then buyers will always exercise the contracts on the day (since $U'(z) > 0$, $\forall z$), and sellers will therefore be forced to deliver the full amount of any option committed with $g = 0$. Such a contract is therefore equivalent to a "must-produce, must-take" contract, i.e., a forward. As we note in Lemma 3, this contract is strictly dominated by an appropriately designed options contract from the seller's perspective without changing the buyer's utility. Thus, interestingly, any such forward contract is Pareto dominated by some options contract when both contract and spot markets are active. Of course, if production costs were lower when supplied under a firm commitment forward contract than under an options contract, then forwards could play a more significant role.[19] However, we see that, absent such cost advantages, pure forwards are not efficient.

A further point of reference is the supply chain coordination literature. Compared with Araman et al. (2002) and Li and Kouvelis (1999), efficiency is achieved here without the use of any penalty for unused contract capacity. Competition, together with proper design of the options contracts, is the underlying efficiency driver. Without competition, options-based contracts may not always coordinate the supply chain (Anand et al. 2003, Cheng et al. 2003, Deng and Yano 2002, Golovachkina and Bradley 2002). With competition, in contrast, we show in Theorem 4 that efficiency does result. Interestingly, even without competition—in the case of one seller and one buyer, Golovachkina and Bradley (2003) show that options contracts can coordinate the supply chain via any of the following three mechanisms: allowing the buyer rather than the seller to be the leader, i.e., the buyer instead of the seller makes offers in the form of $[s, g]$ contracts; or allowing appropriately designed quantity discount; or allowing renegotiation[20]—the seller would match the spot price by charging $g = P_s$ whenever $P_s < g$.

The ultimate driver of our efficiency results is competition in the presence of a backup open spot market.

[17] Thus, we focus only on firms engaged in the contract market, neglecting the consequences of such contracts on spot market allocations, and on firms not engaged in the contract market.

[18] This is the standard characterization for a coordinated supply chain, e.g., Cachon (2003). Note that a first-best solution does not require the buyer or the sellers to break even (although they can be made whole with side payments).

[19] We will discuss this "cost advantage" case further below. Such cost advantages are often argued to result from better production planning, staffing, and maintenance that firm contracts allow. Options contracts can achieve many of these same benefits if the maturation date for calling the contract is sufficiently far in advance of delivery.

[20] This idea of using renegotiation as a channel coordination mechanism has also been explored in Plambeck and Taylor (2005). Note that the contracts arising from our approach are renegotiation proof, because g_i is set equal to b_i, operating cost, and there is no incentive for a seller to accept any lower price on the day than his operating cost.

Further, efficiency is achieved here in companion with other desirable properties such as incentive compatibility and renegotiation proof. This contrasts with other mechanisms discussed above, where channel coordination is at the expense of one or another party's profit. Although our contract market is imperfect, the presence of competition would rule out any monopoly power on this market as long as the heterogeneous technologies are close enough (but not every seller has to be on the leading edge). As long as the sellers' technologies are nearly as efficient as their peers in equilibrium, they enjoy the same benefit as the most efficient seller by selling their entire capacity in advance. This is in contrast with existing B2B models that show a winner-take-all outcome. With competition between as few as two sellers, in equilibrium, no capacity would be wasted—the capacity of seller i ($i = 1, 2$ assume $b_1 < b_2$) either gets called fully on the day (if $P_s > b_i$), or they would produce nothing for the spot market (if $P_s \le b_i$). In either case, each seller collects a net profit of $[p - G(b_i)]K_i$, with the most efficient seller being able to price his options higher. The buyer has nothing to lose either, as the buyer has booked all available capacity at a price less than the expected spot price. In addition, the buyer incurs no unmet demand, thanks to the backup spot market. She can satisfy her entire demand by either procuring partially (say if $b_2 > P_s > b_1$) or entirely (if $P_s \le b_1$) from the spot market.

While competition is clearly the key driver of efficiency, we now discuss factors that influence the structure and proper design of the efficient contracts.

6. Extensions: Portfolio of Contracts, Imperfect Substitutability, and Long-Run Competition

In this section, we illustrate some straightforward extensions based on our modeling framework.[21]

Buyer's Optimal Portfolio of Contracts.[22] Suppose now the buyer is an industrial buyer (who we assume to be risk neutral and profit maximizing). This buyer has a production process that uses only one input, and that input is available through contracting as well as through a B2B exchange. The buyer needs one unit of input for every unit of output. The buyer already owns a certain contract (possibly the buyer owns production assets herself that she can operate at some variable cost) guaranteeing delivery of 100 units of the needed input for the next period's production.

Table 1 **Buyer's Optimal Portfolio of Contracts**

Asset	s	g	$E\{m(P_s - g)^+\}$	Q_i^*
Owned	0	25	6.5	0
Forward	25	0	15	5
Call option 1	20	10	10.7	1.5
Call option 2	22	5	12.7	3.4

The buyer can sell her output in her own product market, and realize revenues $P_s(D(P_s))D(P_s) = (200 - D(P_s))D(P_s)$, where $P_s = 200 - D(P_s)$ is the buyer's inverse demand curve (assumed linear here). Assume the buyer's variable production cost to be 0. If the buyer sources from the seller, then she pays a purchasing cost. Suppose the buyer can either use her existing contract for 100 units or any of the other contracts listed in Table 1 for procuring needed inputs, or alternatively can rely on the spot market. How much of each of these additional contracts should be used, and how much should she rely on her own production asset?

From the theory developed above, the buyer should make the indicated purchases and, on the day, should use these in the order of increasing g. Thus, if the spot market price on the day turns out to be 20, and assume the buyer's computed total demand is 14.6, then the buyer's optimal strategy is to exercise her contracts in the increasing order of g, $5 + 3.4 + 1.5 = 9.9$, and fulfill the residual demand $14.6 - 9.9 = 4.7$ from the spot market. A full pursuit of this idea with applications in electronic power sourcing can be found in Kleindorfer and Li (2005).

Imperfect Substitutability of Contract Spot Purchase.[23] We have so far assumed implicitly perfect substitutability of the contract and spot market good. While reasonable for a variety of generic or commodity goods such as memory chips or steel, perfect substitutability is less appropriate for goods requiring some degree of customization, as this may entail additional costs if not arranged by contract in advance. With slight modifications of the above framework, some forms of imperfect substitutability can be handled easily. We will do so by formalizing the intuitive arguments in Levi et al. (2003) on this matter.

Using the model proposed here, but treating only the case of identical sellers, Levi et al. (2003) consider imperfect substitutability of contract versus spot-purchased goods, both from the buyer's perspective as well as from the supplier's perspective.

In contrast to contract purchases, if the buyer purchases in the spot market, the buyer may incur an additional per-unit "adaptation cost," a, due to the

[21] Additional numerical examples of these extensions—constructed on the basis of this paper—can be found in the survey paper by Kleindorfer and Wu (2003).

[22] This idea has also been explored in Martínez-de-Albéniz and Simchi-Levi (2003a).

[23] We thank an anonymous referee for his/her incisive and constructive comments on this and related issues.

mismatch that might occur between its requirements and the generic product offered in the spot market.[24] The "full price" the buyer pays for spot purchases is therefore "$P_s + a$" instead of "P_s."

The other side of the same coin is from the perspective of the sellers. "Preferred" contract suppliers may enjoy a per-unit production cost (b_c) advantage over "rush-orders-taken" outside suppliers who would have a unit cost of ($b_s \geq b_c$). The difference, $b_s - b_c$, together with the adaptation cost a, represent the costs associated with resolving the imperfect substitutability of contract spot purchase at the time of the spot market with a specific supplier.

To illustrate, suppose that all outside, contract sellers are identical and all produce in the contract market at unit cost b_c and in the spot market at unit cost b_s and that all producers in the spot market also produce at this same unit cost b_s, with $b_c \leq b_s$.[25] The buyer's utility becomes

$$V(q, x; P_s + a, Q) = U\left(\sum_{i=1}^{I} q_i + x\right) - \sum_{i=1}^{I} s_i Q_i - \sum_{i=1}^{I} g_i q_i - (P_s + a)x.$$

We see immediately that Theorem 1 continues to hold. This is true because the buyer's new problem is identical to that modeled in §4.1. by substituting $P_s + a$ for P_s and $G(g_i, a)$ for $G(g_i)$, where $G(v, a)$ is defined as $\mathrm{E}\{\min(P_s + a, v)\}$.

The profit of any seller becomes

$$\pi(s, g, L, P_s + a) = sQ + (g - b_c)q + (P_s + a - b_s)^+ m(P_s, a)(K - q),$$

where $q = Q\chi(P_s + a - g)$ and $m(P_s, a)$ is the probability that any seller can find a spot buyer when the realized spot price is P_s and the adaptation cost is a. Expected profit is therefore given by

$$\begin{aligned} \mathrm{E}\pi(s, g, L; a) &= sQ + (g - b_c)(1 - F(g, a))Q \\ &\quad + (K - Q)\int_{b_c}^{\infty} (P_s + a - b_s) m(P_s, a)\, dF(P_s, a) \\ &\quad + Q\chi(g - b_c)\int_{b_c}^{g} (P_s + a - b_s) m(P_s, a)\, dF(P_s, a), \end{aligned} \quad (11)$$

where $F(v, a)$ denotes the CDF of full spot price $P_s + a$, so that $F(v, a) = \Pr\{P_s + a \leq v\} = F((v - a)^+, 0)$, and $F(v, 0)$ is the CDF of the underlying spot price P_s.

From (11), it is straightforward to obtain the optimal bidding strategies of $L^* = K$ and $g^* = b_c$. Further, Theorems 2 and 3 continue to hold. This is so because the seller's problem remains identical to the one modeled in §4.2., substituting $D(p, a)$ for $D(p)$, $G(b_c, a)$ for $G(b)$, and using the relationship between $G(v, 0)$ and $G(v, a)$, viz., $G(v, a) = \min[v, G((v - a)^+, 0) + a]$.

The introduction of imperfect substitutability strengthens our findings in several ways. First, the increase in b_s over b_c reflects the additional cost to the last-minute seller of meeting the buyer's requirements, making contracting even more attractive from the seller's perspective. Indeed, it is straightforward to show that contracting cost advantages can make contracting (including forwards) desirable from the sellers' perspective even when $m(P_s, a) = 1$, in contrast to the base case where no such cost advantages are present. Second, the introduction of additional spot sourcing cost on the buyer side implies indirectly that $m(P_s, a)$ may decrease, which, together with the direct cost increase $P_s + a$, makes the spot market a less viable alternative to contracting for the buyer. In either case, both the buyer and sellers would rely more on contracting sourcing—enhancing previous results. We have discussed elsewhere managerial implications and other extensions, empirical evidence, as well as testable hypotheses on predictions for electronic markets (Levi et al. 2003).

Long-Run Technology Choice. What happens in the long run when capacity K is also a choice variable? Wu et al. (forthcoming) note that the above short-term results can be generalized to capture the nature of long-term equilibria, in which sellers can adjust capacity, anticipating the payoffs they will receive from the fixed capacity game based on this paper. This analysis characterizes the nature of efficient technology mixes that are likely to survive in a long-run equilibrium when firms with heterogeneous cost structures compete in a rich contract spot market setting. Essentially, the separating equilibrium results as reported in Remark 3 in §4.3. go through with the exception that in the long run, some firms will exit the market.

7. Summary and Discussion

This paper models the interaction between capacity, contracting, and output decisions. The results are strikingly different from the case of a single buyer-seller

[24] As envisioned by a referee, this adaptation cost captures the "reductions in utility," perhaps related to performance or quality risks, when transacting with outside suppliers in the spot market. Or, again as suggested by the referee, such adaptation costs may also arise from extra costs the buyer has to absorb when transacting with outside suppliers due to "costs of coordination, settlement, and integration with new suppliers."

[25] The more general case where contractors are heterogeneous or where contractors possess more efficient technologies versus outside sellers awaits a detailed analysis. Intuitively, one would expect that the results presented here would only be strengthened in the event that some contractors possess more efficient technologies for spot production than available in the general spot market. In any case, these generalizations complicate the analysis considerably.

relationship. Indeed, relative to our own previous work and to that of the OM literature generally, the element of competition and its interaction with heterogeneous cost and capacity-endowed sellers is the key contribution of this paper.

The model derives the optimal use of forwards, options, and spot market procurement in the presence of competition among multiple sellers with heterogeneous technologies. This paper characterizes the structure and sufficient existence conditions for an integrated contract-spot market equilibrium with multiple sellers and a buyer. This yields analytic results on the pricing of capacity options and the value of managerial flexibility, as well as providing conditions under which B2B exchanges are efficient and sustainable as market-clearing platforms for contracting and spot purchases.

Three indices are important to understand the results of this paper. First, the contract index $s + G(g)$ is used by the buyer to screen and order contracts ex ante. Second, on the day, the option execution fee g is used by the buyer to order the execution of contracts. Third, the $c = \underline{s} + G(b) = \mathrm{E}\{m(P_s)(P_s - b)^+\} + G(b)$ index is the key for a seller to participate in the contract/forward market in preference to participation in the spot market. These indices do not necessarily follow the same rank order, and each is central to a particular operational decision. The integration of these decisions under competitive conditions yields new insights on the optimal mix of diverse technologies that are likely to survive in competitive equilibrium.

We now discuss the role of the assumptions played in our model.

Contract Market. Modeled here is an imperfect forward market, i.e., only a small number of prequalified sellers can participate in this market, while there are outside sellers who are precluded from participating in the contract market, although they can participate in the spot market. As Allaz and Vila (1993) point out, the standard literature in financial economics is not realistic in its assumption of perfect forward markets. Especially in capital-intensive industries (e.g., Wilson 2002), there are many factors that call for a restricted contractor base. The causes of this are captured in rich detail in our model.

Notwithstanding the practical relevance of our assumptions on imperfect market structure, there must be some relationship between the average spot price and the price and costs obtained in the contract market (Slade and Thille 2002). Indeed, as Theorem 2 shows, the contract price will be less than the expected spot price, and this is observed in numerous markets (e.g., Slade and Thille 2002). Thus, it is something of a fiction to imagine that just any subset of sellers has been selected by a buyer to participate in the set of eligible contract sellers. One would imagine, for example, that if a very inefficient seller (relative to those participating in the spot market) were selected, screened, and qualified for participation in this market, then such a seller would either end up supplying very little or perhaps, after a while, would end up being completely excluded from contract purchases. This is, in fact, what also results in our model. A relatively less efficient seller (one with a higher operating cost b) will indeed end up with a smaller share (and perhaps a zero share) of the equilibrium allocation. The seller's contract allocation in equilibrium would be very small if the seller had operating cost significantly in excess of the average spot market price. Exactly how well such a seller would fare depends, as we showed, on the competitive access conditions and variability of spot price. In a very volatile market, the options provided by such a seller, even when b is in excess of average spot price, can be valuable. All of this, in our view, links well to reality and to the intuition about the general relationship between participants in the contract market and participants in the spot market.[26]

Risk of Last-Minute Sales (m). What does it mean to assume $m < 1$? The basic rationale for this in practice is that finding appropriate buyers at the last minute does not always work, even when a seller is willing to produce at lower than the prevailing spot price. "Appropriate" might mean having access to transmission or transportation, which may not be available last minute. It might mean that the buyer requires infeasible, last-minute customization. It might mean inability to assure credit worthiness. Any of these could contribute to less than perfect access to the spot market, as captured in $m < 1$.

This market access probability function will be determined by different factors in different market settings. In electric power, access will depend on transmission constraints, e.g., where certain sellers may face significant barriers to their participation in the spot market, as dispatched by the independent system operator, because of transmission constraints. This condition often leads to locally "thin spot markets" (e.g., Wilson 2002). Clearly, goods or services transacted in such spot markets are nonstorable and cannot be "rolled over." What happens if $m = 1$? Under perfect substitutability, as we have shown, any seller who enjoys perfect market access ($m = 1$) will find no reason to contract, because risk-neutral buyers can purchase all they need from the spot market and will therefore not pay more than μ. Other reasons for contracting (even when $m = 1$) include imperfect substitutability (as captured by positive adaptation costs $a > 0$), risk aversion by sellers or buyers, and lower production costs in the contract market than

[26] We thank a referee for drawing our attention to this.

in the spot market (e.g., Golovachkina and Bradley 2002, 2003; Lee and Whang 2002). A margin loss of α percentage of the seller or a margin requirement by the buyer, as assumed by these authors, is precisely equivalent to, and has the same effect as, $m = 1 - \alpha$ in the present framework.

As an example, consider the hotel business (e.g., Quan 2002). Here, access depends on booking systems, especially those that clear the market on or close to the day of occupancy. The concern by hoteliers (as sellers) is that they may end up with unoccupied rooms even when there is a spot market for last-minute reservations. The fact that not every hotel has access to every buyer participating in the spot market is a strong reason for the hotel to sell in advance reservations to both individuals and convention planners/consolidators. In our model, note that $m(P_s)$ is a function of the spot price P_s and could very well satisfy $m = 1$ for many states of the world. What is usually the case, and what we allow for here, is that when the spot price is very high, reflecting both scarcity of underlying supply as well as near-excess demand conditions, the marginal seller and demander may have some difficulties matching up with one another. In particular, note that in real markets in capital-intensive industries, most supply is done under contract, with only fine-tuning left to last-minute adjustments. The reason often noted is that sellers are concerned about finding last-minute buyers for their capacity in a volatile spot market (as allowed for here). In addition, the presence of imperfect substitutability of contact versus spot purchase would push the trade of options-based contracts, even if the seller has perfect spot access. This can easily be seen using the measure $(p - G(b_c, 0))/(\mu - G(b_s, a))$ (cf. Remark 1 in §4.3.) of spot "illiquidity." An increase of a, or b_s over b_c, would increase this threshold, therefore widening the trade spread of contracting over spot transactions.

Spot Market. We assume here an open spot market, i.e., no particular seller has market power in the spot market. This is a crucial assumption in the modeling because if a seller can control the spot market (as in the case of a closed spot market with a dominant player), that seller might very well withhold supply in the contract or spot market in order to drive up the spot price.[27] This very scenario has been observed in electricity markets, though it appears not to require very much competition to drive spot prices to nearly competitive levels (Wolfram 1999, Wilson 2002). What one would expect in the above framework if the spot market has the same players as the contract market (consisting of a small number of competing sellers), then one would expect more contract-intensive behavior from integrating the two markets, with or without options. In such a setting, the results of Allaz and Vila (1993) and Mendelson and Tunca (2004a, b) suggest that sellers would be worse off by establishing a B2B exchange to facilitate their contracting and spot sales. This remains a conjecture, as the closed-market, multiseller case with options and heterogeneous costs remains "open."

[27] For modeling of this type of strategic use of inventory and its impact on options contracts efficiency, see Anand et al. (2003).

Future Research. Our equilibrium results (Theorem 2) show that all capacity is contracted for in equilibrium, and contrasts with existing single-seller models, e.g., Wu et al. (2002). However, this behavior is often not observed in practice, where sellers end up selling both in the contract market and in the spot market. Why is this? One reason is, perhaps, due to nonrisk neutral preferences by seller decision makers, which could drive them to speculate on high returns in the spot market. Another reason, as explored in Spinler et al. (2003) is state-dependent demand or cost conditions. Another reason could be that production costs are nonlinear and increasing as output approaches capacity limits. This would give rise to producing in a contract market at normal output levels and selling to the spot market only when the spot price is high (enough to warrant higher marginal cost production). Of course, a multiplant facility, having plants with different cost characteristics, could very well have some plants participate in the contract market and others in the spot market. Generalizing the current framework to accommodate such a multifacility company model (or more generally, nonconstant marginal cost bidding model) remains an open question. These issues, and their empirical validation and use in developing applications, await future research.

To summarize, options contracts of the form discussed here are of increasing importance in practice and are having fundamental impacts on both B2B contracting as well as the operational decisions that flow from it. Naturally, there is still much to learn about the nature of individual B2B markets and the manner in which such options contracts are being shaped to serve these markets.

An electronic companion to this paper is available at http://mansci.pubs.informs.org/ecompanion.html.

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Appendix. Notation

Ξ: the set of sellers

P_s: spot market price, with cumulative distribution function $F(P_s)$ and density function $f(P_s)$, where μ is the mean of the spot market price

b: seller's short-run marginal cost of providing a unit

K: seller's total capacity

L: seller's posted capacity to the contract market

s: seller's reservation cost per unit of capacity

g: seller's execution cost per unit of output

$\pi(s, g, L, P_s)$: seller's entire profit from both the contract market and the spot market given a realized spot market price P_s and other sellers' bid

Q: buyer's contract vector for seller's output

q: buyer's contract consumption of seller's output

x: buyer's spot market purchase

$U(z)$: buyer's aggregate WTP for output level z

$V(q, x; P_s, Q)$: buyer's utility

$m(P_s)$: The probability vector that sellers can find a last-minute buyer on the spot market when the realized spot price is P_s

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